

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/347051422>

# Co-Even Domination Number in Some Graphs

Article in IOP Conference Series Materials Science and Engineering · November 2020

DOI: 10.1088/1757-899X/928/4/042015

CITATION

1

READS

8

2 authors:



**Manar Makki**

University of Babylon

2 PUBLICATIONS 2 CITATIONS

[SEE PROFILE](#)



**Ahmed A Omran**

University of Babylon

21 PUBLICATIONS 24 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Domination in discrete topology graph [View project](#)



DOMINATION IN RHOMBUS CHESSBOARD [View project](#)

PAPER • OPEN ACCESS

## Co-Even Domination Number in Some Graphs

To cite this article: Manar M. Shalaan and Ahmed A. Omran 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **928** 042015

View the [article online](#) for updates and enhancements.

**The 17th International Symposium on Solid Oxide Fuel Cells (SOFC-XVII)**  
**DIGITAL MEETING • July 18-23, 2021**

**EXTENDED Abstract Submission Deadline: February 19, 2021**



**SUBMIT NOW →**

## Co-Even Domination Number in Some Graphs

Manar M. Shalaan<sup>1</sup> and Ahmed A. Omran<sup>2</sup>

<sup>1&2</sup>Department of Mathematics, College of Education for Pure Science, University of Babylon, Babylon, Iraq

<sup>1</sup>[manarmacki92@gmail.com](mailto:manarmacki92@gmail.com)

<sup>2</sup>[pure.ahmed.omran@uobabylon.edu.iq](mailto:pure.ahmed.omran@uobabylon.edu.iq)

### Abstract

The purpose of this work is to determine the co-even domination number of various graphs, as a ladder, lollipop, butterfly, jellyfish, helm, corona, fan, and double fan graph. Before this, the important properties of the co-even dominating set are mentioned from previous work.

**Keywords:** Co-even dominating set, Co-even domination number, Lollipop, Butterfly, jellyfish, Helm, Ladder.

### 1. Introduction

Domination is one of the most important notions in graph theory because it is an important way to solve many problems in life. Recently, the notion of domination is interested in many researchers whether from researchers of mathematics, engineering, chemistry, or biology, and others. There are many kinds of domination are discussed [1], [2]. Also, it is introduced in many fields of graph theory as labeled graph [3,4] and topological graph [5], and the others. This work is an extension of previous work as researchers Ahmed A. Omran and Manar M. Shalaan [6] presented a new definition of domination which is called co-even domination and definition as follows, assume that  $G$  be a graph and  $D$  is a dominating set, the set  $D$  is called co-even dominating set if  $v$  has even neighbors for all  $v \in V - D$ . This set denoted by  $CEDS$ , also if it has no proper  $CEDS$ , then it is called minimal  $CEDS$  and denoted by  $MCEDS$ . Furthermore, if  $MCEDS$  has minimum cardinality, then it is called co-even domination number and denoted by  $\gamma_{coe}(G)$ . In this work, this number is determined by various graphs. The first graph is the ladder graph that obtained by Cartesian product of  $P_2$  and  $P_n$ . The second graph is called fan that defined by join the path and complete of order one and denoted by  $F_n \equiv P_n + K_1$ . The third graph is called double fan which obtains by  $DF_n \equiv P_n + \overline{K_2}$ . The fourth graph, obtained by the operation corona to two graphs one of them the cycle graph of order  $n$  and the other is a null graph of order  $p$  and denoted by  $C_n \odot \overline{K_p}$ . The fifth graph is called helm that obtained from the wheel of  $n$  vertices by adjoining a pendent edge at each vertex of the cycle. The sixth graph is the lollipop which obtained from a complete  $K_m$  graph with a single vertex attached by a path of  $n$  - length and denoted that  $L_{m,n}$ .

The seventh graph is the butterfly and defined as follows, let  $C_n$  be a graph of order  $n$ , the vertex common  $(n - 3)$  chords is called the apex. A subgraph induced by apex with end vertices of



chords is called a shell graph. A butterfly graph is a double shell for any orders  $(m, n)$  with common apex with exactly two pendant edges at the apex vertex. Finally, the eighth graph is The jellyfish which obtained from a cycle of order 4 with vertices  $v_1, v_2, v_3,$  and  $v_4$  by joining  $v_1$  and  $v_3$  with an edge and appending  $m$  pendant edges to  $v_2$  and  $n$  pendant edges to  $v_4$ . The reader can be found more details about the definitions above in [7-11].

**Proposition 1.1.[6],** Let  $G = (n, m)$  be a graph and  $D$  is a co-even dominating set, then

1) All vertices of odd or zero degrees belong to every co-even dominating set.

2)  $deg(v) \geq 2, \text{ for all } v \in V - D$

3) If  $G$  is  $r$ -regular graph then  $\gamma_{coe}(G) = \begin{cases} n, & \text{if } r \text{ is odd} \\ \gamma(G), & \text{if } r \text{ is even} \end{cases}$

4)  $\gamma(G) \leq \gamma_{coe}(G)$ .

**Proposition 1.2. [6],** Let  $G$  be a path of order  $n$ , so  $\gamma_{coe}(G) = 2 + \lceil \frac{n-4}{3} \rceil$ .

## 2. Main results

**Proposition 2 .1.** if  $G$  is a Cartesian product of  $P_2$  and  $P_n$  denoted by

$$G \equiv P_2 \times P_n, \text{ then } \gamma_{coe}(G) = 2n - 4.$$

**Proof .**let  $G$  be the Cartesian product of two paths of order  $n$ , so  $G$  of order  $2n$  as shown in Figure 2.1. One can easily show that each vertex of  $G$  has an odd degree except four vertices that represent the end vertices of each path which has even degree. Thus, according to proposition 1.1(1), all vertices have odd degrees belong to every co-even dominating set and these vertices dominate the remain four vertices. Therefore,  $\gamma_{coe}(G) = 2n - 4$ .

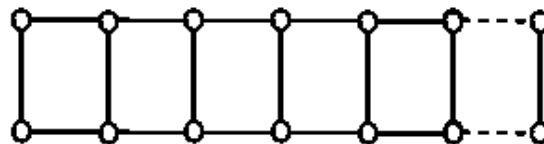


Figure 2.1.  $P_2 \times P_n$

**Proposition 2 .2.** if  $G$  is the Fan graph of order  $n + 1$  is defined by

$$G \equiv F_n \equiv P_n + K_1, \text{ then } \gamma_{coe}(G) = \begin{cases} n - 2, & \text{if } n \text{ is even} \\ n - 1, & \text{if } n \text{ is odd} \end{cases}$$

**Proof .** let  $G$  be a fan graph of order  $n + 1$ , then every vertex of  $P_n$  joining with  $K_1$ , so there exist two cases are discussed as follows.

**Case1.** if  $n$  is even, then every vertex of  $G$  has an odd degree except two end vertices in  $P_n$  and the vertex of  $K_1$  as shown in Figure 2.2. Thus, according to proposition 1.1(1), all

vertices of degree odd belong to co-even dominating set, and these vertices dominate the other vertices. Therefore,  $\gamma_{coe}(G) = n - 2$ .

**Case2.** If  $n$  is odd, then every vertex in  $G$  has an odd degree except two end vertices in  $P_n$  have even degree. Again, by proposition 1.1(1), one can easily prove that  $\gamma_{coe}(G) = n - 1$ .

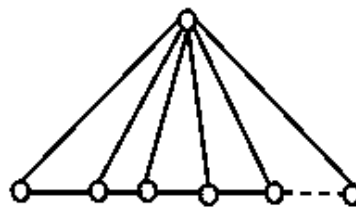


Figure 2.2.  $P_n + K_1$

**Proposition 2.3.** if  $G$  is the double Fan of order  $n+2$  is defined by

$$F_n \equiv P_n + \overline{K_2}, \text{ then } \gamma_{coe}(G) = \begin{cases} 2, & \text{if } n \text{ is even; } n \leq 4 \\ 3, & \text{if } n \text{ is even; } n > 4 \\ 4, & \text{if } n \text{ is odd} \end{cases}$$

**Proof.** By definition of join two vertices all vertices in the path adjacent to the two vertices out of this path which represents of the graph  $\overline{K_2}$  as shown in Figure 2.3. Thus, there are two different cases as follows.

**Case 1.** If the number of vertices in the path is even, then all vertices in  $F$  have even degree except the end vertices of path. These two vertices belong to every co-even dominating set, according to proposition 1.1(1), and dominate other vertices if the order of path is less than or equal four, so in this case  $\gamma_{coe}(G) = 2$ . Otherwise, these vertices do not dominate some of the vertices which belong to the path in  $F$ , thus in this case  $\gamma_{coe}(G) \geq 3$ . Now, if one of the two vertices that represent  $\overline{K_2}$  is chosen with the previous two vertices, the three vertices are dominating all other vertices. Therefore, in this case  $\gamma_{coe}(G) = 3$ .

**Case 2.** If the number of vertices in path is even, as same manner in case1, all vertices in  $F$  have even degree except the four vertices which are the end vertices of path and the two vertices of  $\overline{K_2}$ . According to proposition 1.1(1), these four vertices belong to every CEDS and they are dominate all other vertices. Thus,  $\gamma_{coe}(G) = 4$ .

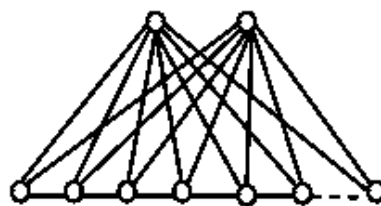


Figure2.3.  $P_n + \overline{K_2}$

**Proposition 2.4.** if  $G \equiv C_n \odot \overline{K_p}$ , then  $\gamma_{coe}(G) = \begin{cases} np, & \text{if } p \text{ is even} \\ np + n, & \text{if } p \text{ is odd} \end{cases}$ .

**Proof.** In the graph  $G$  shown in Figure 2.4, two cases are discussed as follows.

**Case1.** If  $p$  is even, then the degree of all vertices of  $C_n$  are even. Therefore, by proposition 1.11(1), all pendent vertices of  $G$  belong to every co-even dominating set and these vertices dominate the vertices of the cycle. Thus,  $\gamma_{coe}(G) = np$ .

**Case2.** If  $p$  is odd, then the degree of all vertices of  $G$  are odd. Therefore, according to proposition 1.1 (1),  $\gamma_{coe}(G) = np + n$ .

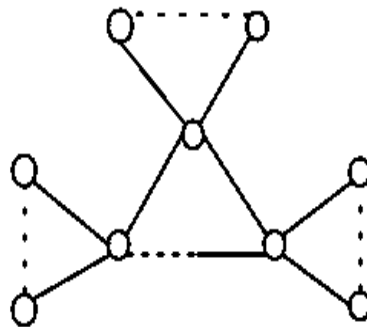


Figure 2.4.  $C_n \odot \overline{K_p}$

**Proposition 2.5.** If  $G \equiv H_n$  be helm graph of  $(2n - 1)$  vertices ,then

$$\gamma_{coe}(G) = n.$$

**Proof.** Let  $G$  be helm graph of order  $(2n - 1)$ , then the number of vertices in induced subgraph isomorphic to cycle is  $(n - 1)$  as shown in Figure 2.5. All these vertices have even degree since each vertex of them adjacent to two vertices in the cycle, one pendant vertex, and the center of wheel graph. By proposition 1.1(1), all pendant vertices of  $G$  belong to every co-even dominating set and these vertices dominate all vertices in  $G$  except the center of the wheel. Therefore,  $\gamma_{coe}(G) = n$

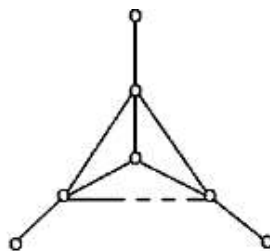


Figure 2.5. helm graph  $H_n$

**Proposition 2.6.** if  $G$  is a Lollipop graph  $L_{m,n}$ , then

$$\gamma_{coe}(G) = \left\{ \begin{array}{ll} 2, & \text{if } m \text{ is odd and } n \leq 3 \\ 3 + \left\lceil \frac{n-6}{3} \right\rceil, & \text{if } m \text{ is odd and } n \geq 4 \\ m, & \text{if } m \text{ is even and } n = 2 \\ m + 1 + \left\lceil \frac{n-5}{3} \right\rceil, & \text{if } m \text{ is even and } n \geq 3 \end{array} \right\}, \text{ where } m \geq 3, n \geq 2$$

**Proof.** From the definition of Lollipop graph  $L_{m,n}$ , one can easily conclude that the vertex in the induced complete subgraph say  $v_1$  which adjacent to a vertex in induced path subgraph say  $u_1$ , the degree of the vertex  $v_1$  is different from the degrees of other vertices in the induced complete subgraph since  $v_1$  is adjacent to  $u_1$  as shown in Figure 2.6. To get to the results, there are two cases are discussed as follows.

**Case 1.** If the number of vertices in induced complete subgraph ( $m$ ) is odd, then all these vertices have even degree except the vertex  $v_1$ . Therefore, the vertex  $v_1$  belongs to every  $MCEDS$ , and it is obvious that this vertex dominates all vertices in the induced complete subgraph. So, there are two subcases as follows.

**Subcase 1.** If  $n \leq 3$ , then the number of vertices in the induced path subgraph is either two or three. If  $n = 2$ , then the vertices of the induced path subgraph are  $\{u_1, u_2\}$ , the vertex  $u_2$  belongs to every  $MCEDS$  according to proposition 1.1(1). Also, If  $n = 3$ , then the vertices of the induced path subgraph are  $\{u_1, u_2, u_3\}$ , the vertex  $u_3$  must belongs to every  $MCEDS$  according to proposition 1.1(1). Therefore,  $\gamma_{coe}(G) = 2$ .

**Subcase 2.** If  $n > 3$ , then the vertex  $v_1$  is dominating the vertex  $u_1$ , so to choose the  $MCEDS$ , must  $u_3$  taken in it and by proposition 1.1(1), the vertex  $u_n$  belongs to every  $MCEDS$ . Thus, according to proposition 1.2,  $\gamma_{coe}(G) = 3 + \left\lceil \frac{n-6}{3} \right\rceil$ .

**Case 2.** If the number of vertices in induced complete subgraph ( $m$ ) is even, then all these vertices have odd degree except the vertex  $v_1$ . According to proposition 1.1(1), all these vertices belong to every  $MCEDS$ . There are two subcases.

**Subcase 1.** If  $n = 2$ , then again the pendant vertex  $u_2$  belongs to every  $MCEDS$ . Therefore,  $\gamma_{coe}(G) = m - 1 + 1 = m$ .

**Subcase 2.** If  $n \geq 3$ , then the remain vertices not dominated by  $m - 1$  vertices of induced complete subgraph represent the induced path subgraph, the vertex  $u_2$  dominates the vertex  $u_1$ , and the vertex  $u_n$  belongs to every  $MCEDS$  by proposition 1.1(1). Thus, according to proposition 1.2,  $\gamma_{coe}(G) = m - 1 + 2 + \left\lceil \frac{n-5}{3} \right\rceil = m + 1 + \left\lceil \frac{n-5}{3} \right\rceil$ .

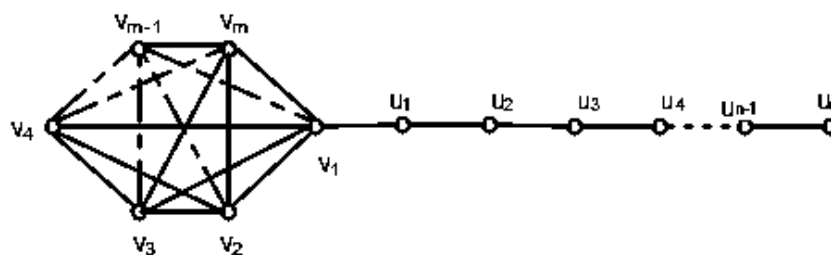


Figure 2.6. lollipop graph  $L_{m,n}$

**Proposition 2.7.** if  $G$  is a butterfly graph  $BF(m, n)$ , then

$$\gamma_{coe}(G) = \left. \begin{matrix} m + n - 2, & \text{if } m, n \text{ are even (odd)} \\ m + n - 1, & \text{otherwise} \end{matrix} \right\}, \text{ where } m, n \geq 2.$$

**Proof.** Let  $G$  be a butterfly graph with order shells  $m$  and  $n$  except the apex and have  $(m + n + 3)$  vertices with edges  $2(m + n)$ . The vertices of wings of the butterfly graph are  $v_1, v_2, \dots, v_m$  and  $u_1, u_2, \dots, u_n$  and the pendant vertices  $v$  and  $u$  as shown in Figure 2.7. Thus, by proposition 1.1(1)  $\{v, u\}$  and the set  $\{v_2, \dots, v_{m-1}, u_2, \dots, u_{n-1}\}$  belong to every co-even dominating set. Now there exists two cases are discussed as follows.

**Case1.** If  $m$  and  $n$  are even (odd), then the degree of apex vertex say  $w$  is even, therefore

$$\begin{aligned} \gamma_{coe}(G) &= \gamma_{coe}(P_m) + \gamma_{coe}(P_n) + \{v, u\} = (m - 2) + (n - 2) + 2 \\ &= m + n - 2 \end{aligned}$$

**Case2.** If  $m$  or  $n$  is odd, then the degree of  $w$  is odd, therefore

$$\begin{aligned} \gamma_{coe}(G) &= \gamma_{coe}(P_m) + \gamma_{coe}(P_n) + \{v, u, w\} = (m - 2) + (n - 2) + 3 \\ &= m + n - 1 \end{aligned}$$

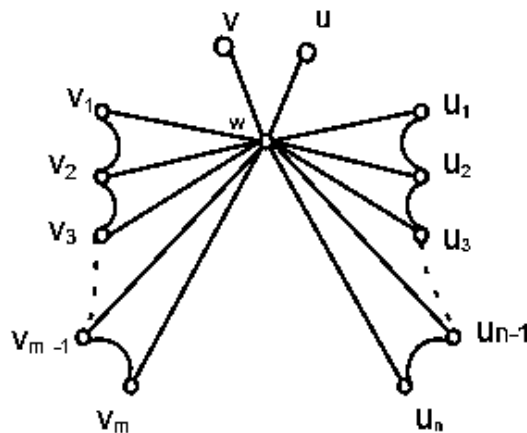


Figure 2.7. Butterfly graph  $BF(m, n)$

**Proposition 2.8.** if  $G$  is a jellyfish graph  $J(m, n)$ , then

$$\gamma_{coe}(G) = \left. \begin{matrix} m + n + 2, & \text{if } m, n \text{ are even} \\ m + n + 4, & \text{if } m, n \text{ are odd} \\ m + n + 3, & \text{if either } m \text{ or } n \text{ is even} \end{matrix} \right\}, \text{ where } m, n \geq 1$$

**Proof.** Let  $G$  be jellyfish graph on  $(m + n + 4)$  vertices and  $(m + n + 5)$  edges, where  $m, n \geq 1$  as shown in Figure 2.8. Then there are three cases that depend on  $m$  and  $n$  as follows.

**Case1.** if  $m$  and  $n$  are even, then there exist two vertices say  $\{v, u\}$  are joining with  $m$  and  $n$  have even degree, by proposition 1.1 (1) all vertices of  $G$  except  $\{v, u\}$  belongs to every co-even dominating set, therefore  $\gamma_{coe}(G) = m + n + 2$ .

**Case2.** if  $m$  and  $n$  are odd, then every vertex of  $G$  have odd degrees. Therefore, by proposition 1.1 (1) all vertices of  $G$  belong to every  $MCEDS$ . Hence,  $\gamma_{coe}(G) = m + n + 4$ .

**Case3.** It is obvious.

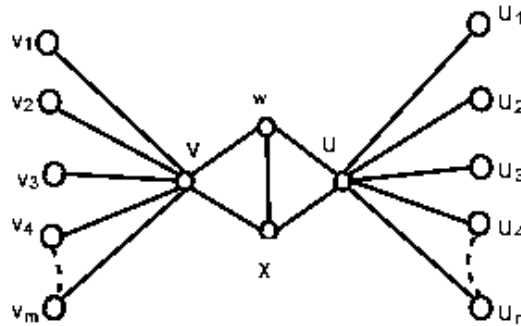


Figure2.8. jellyfish graph  $J(m, n)$

### 3. Conclusion

Through, this work, the co-even domination number of some kinds of graphs is determined, These graphs are a ladder,  $C_n \odot \overline{K_p}$ , lollipop, butterfly, jellyfish, helm, fan and double fan graph.

### References

- [1] A. A. Omran and Haneen H. Oda, "Hn-Domination in Graphs." Baghdad Science Journal 16.1, 242-247, (2019).
- [2] A. A. Omran and Y. Rajihy, Some properties of frame domination in graphs, Journal of Engineering and Applied Sciences, 12 (2017), 8882-8885.
- [3] M.N. Al-Harere, A. A. Omran, On binary operation graphs, Boletim da Sociedade Paranaense de Matematica, Vol 38 No7, 59-67,2020.
- [4] M.N. Al-Harere, A. A. Omran, Binary operation graphs, AIP conference proceeding vol.2086, Maltepe University, Istanbul, Turkey,030008, 31 July - 6 August (2018). <https://doi.org/10.1063/1.5095093>.
- [5] A. A. Jabor., A. A. Omran., Domination in Discrete Topology Graph, AIP. Third International Conference of Science(ICMS2019),Vol.2183(2019): 030006-1–030006-3; <https://doi.org/10.1063/1.5136110>.
- [6] M.M. Shalaan and A. A. Omran, Co-even Domination in Graphs, International Journal of Control and Automation, Vol. 13, No. 3, pp. 330-334, 2020.
- [7] Ando, Kiyoshi (2007), "Contractible edges in a k-connected graph", Discrete geometry, combinatorics and graph theory, Lecture Notes in Comput. Sci., 4381, Springer, Berlin,
- [8] F. Harary, Graph Theory, Addison-Wessley, Reading Mass. (1969).
- [9] T. W. Haynes, S. T. Hedetniemi and P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., New York (1998).
- [10] Koh, Khee Meng, et al. Graph Theory: Undergraduate Mathematics. World Scientific Publishing Company, 2015.
- [11] Jonasson, Johan ,"Lollipop graphs are extremal for commute times". Random Structures and Algorithms. 16 (2): 131–142, (March 2000).