Solutions Manual for

ELECTROMAGNETISM:
PRINCIPLES AND APPLICATIONS

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Contents

NOTES

1. So as to save space, simple mathematical expressions are typed on a single line. Whenever the order of the operations is not indicated explicitly by means of parentheses, they are performed in the following order:

multiplications,
divisions,
additions and subtractions.

Examples:

$$1/ab = \frac{1}{ab}\ ,\quad 1/a + b = \frac{1}{a} + b\ ,\quad 1/(a+b) = \frac{1}{a+b}\ ,$$

$$1 + a/b = 1 + \frac{a}{b},\quad 1/(a+b/c + de/fg) = \frac{1}{a + \frac{b}{c} + \frac{de}{fg}}\ .$$

2. Programs for drawing the curves in this Manual with a HP9820 calculator and a 9862A plotter are available free of charge from the undersigned.

3. Reference is made, occasionally, to a table of integrals by Dwight. The full reference is "Tables of Integrals and Other Mathematical Data" by Herbert Bristol Dwight (Macmillan).

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CHAPTER 1

1-1 (1.2)

$\vec{A}.\vec{B} = AB\cos\theta = 9 \times 4 - 6 - 30 = 0$

1-2 (1.2)

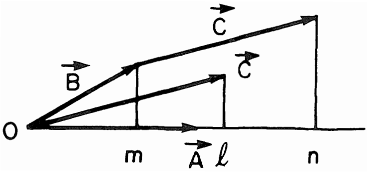
$\vec{A}.\vec{B} = AB\cos\theta = 2 - 18 + 1 = -15; \quad AB = (4+9+1)^{\frac{1}{2}}(1+36+1)^{\frac{1}{2}} = 23.1$

$\cos\theta = -0.650, \ \theta = 130.5^{\circ}$

1.3 (1.2)

$\vec{A}.(\vec{B}+\vec{C}) = A\,\overline{0n},$

$\vec{A}.\vec{B} + \vec{A}.\vec{C} = A\,\overline{0m} + A\overline{0\ell} = A\,\overline{0n}$



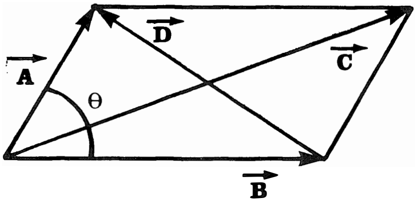
1-4 (1.2)

$C^2 = \vec{C}.\vec{C} = \vec{A}.\vec{A} + \vec{B}.\vec{B} + 2\vec{A}.\vec{B}$

$= A^2 + B^2 + 2AB\cos\theta$

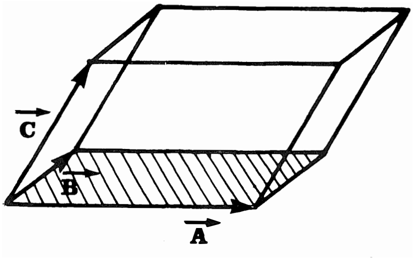
$D^2 = A^2 + B^2 - 2AB\cos\theta$

$C^2+D^2 = 2(A^2+B^2), \ C^2 - D^2 = 4AB\cos\theta$



1-6 (1.3)

$\vec{A} \times \vec{B}$ is normal to the plane of $\vec{A}$ and $\vec{B}$. Its magnitude is the area shown hatched. Then $|(\vec{A} \times \vec{B}).\vec{C}|$ is the base of the parallelepiped, multiplied by its height, or its



volume. Similarly, $\vec{A}.(\vec{B} \times \vec{C})$ is also the volume of the parallelepiped.

<u>1-7</u> (1.3)

The x-component is $A_y(B_z+C_z) - A_z(B_y+C_y) = (A_yB_z-A_zB_y) + (A_yC_z-A_zC_y)$, or the x-component of $\vec{A} \times \vec{B} + \vec{A} \times \vec{C}$. The same applies to the y- and z- components.

<u>1-8</u> (1.3)

For the x-component,

$$a_y(b_xc_y-b_yc_x) - a_z(b_zc_x-b_xc_z) = b_x(a_xc_x+a_yc_y+a_zc_z) - c_x(a_xb_x+a_yb_y+a_zb_z)$$

The corresponding equations for the y- and z- components can be found by rotating the subscripts.

<u>1-9</u> (1.4)

$d\vec{r}/dt$ is perpendicular to $\vec{r}$. Then r is a constant. Also,

$$(d/dt)(\vec{r}.\vec{r}) = 2\vec{r}.(d\vec{r}/dt) = 0$$

$$= (d/dt)r^2 = 2r(dr/dt)$$

Then $dr/dt = 0$ and $r =$ constant.

<u>1-10</u> (1.4)

$$x = 500(\cos 30^o)t\text{ , } y = 500(\sin 30^o)t-4.90t^2$$

$$= 433t \qquad = 250t - 4.90t^2$$

$$\vec{r} = 433\,t\,\vec{i} + (250t - 4.90t^2)\vec{j},$$

$$\vec{v} = 433\,\vec{i} + (250 - 9.80t)\vec{j},$$

$$\vec{a} = -9.80\,\vec{j}$$

<u>1-11</u> (1.5)

$$\nabla(\vec{A}.\vec{r}) = \nabla(A_x x+A_y y+A_z z) = (\partial/\partial x)(\quad)\vec{i}+(\partial/\partial y)(\quad)\vec{j}+(\partial/\partial z)(\quad)\vec{k}=\vec{A}$$

<u>1-12</u> (1.5)

$$(\vec{A}.\nabla)\vec{r} = \left[A_x(\partial/\partial x)+A_y(\partial/\partial y)+A_z(\partial/\partial z)\right](x\vec{i}+y\vec{i}+3\vec{k}) = \vec{A}$$

<u>1-13</u> (1.5)

a) $\nabla'(1/r) = \vec{i}(\partial/\partial x')(1/r) + \vec{j}(\partial/\partial y')(1/r) + \vec{k}(\partial/\partial z')(1/r)$

where $r = \Big[(x'$

Now $(\partial/\partial x')(1/r) = -(1/r^2)(\partial r/\partial x') = -(1/r^2)(x'-x)/r = (x-x')/r^3$

By symmetry, $(\partial/\partial y')(1/r) = (y-y')/r^3, (\partial/\partial z')(1/r) = (z-z')/r^3$

Since $x-x'$ is the x-component of $\vec{r}$, and $(x-x')/r$ is the x-component of $\vec{r}_1$, etc, $\nabla'\cdot(1/r) = \vec{r}_1/r^2$

b) In this case, $\nabla(1/r) = \vec{i}(\partial/\partial x)(1/r) + \vec{j}(\partial/\partial y)(1/r) + \vec{k}(\partial/\partial z)(1/r)$

$(\partial/\partial x)(1/r) = -(1/r^2)(\partial r/\partial x) = -(1/r^2)(x-x')/r = -(x-x')/r^3$ and similarly for the other derivatives. Then $\nabla(1/r) = -\vec{r}_1/r^2$

<u>1-14</u> (1.5)

a)
$$\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ \partial f/\partial x & \partial f/\partial y & \partial f/\partial z \end{vmatrix} = \vec{r} \times \nabla f$$

b) $\vec{A}.\vec{r} = (\vec{r} \times \nabla f)\cdot\vec{r}$ is zero, since $\vec{r} \times \nabla f$ is perpendicular to $\vec{r}$.

c) $\vec{A}\cdot\nabla f$ is zero for the same reason.

<u>1-15</u> (1.8)

a) $\nabla.\vec{r} = (\partial/\partial x)x + (\partial/\partial y)y + (\partial/\partial z)z = 3$

b) The flux of $\vec{r}$ is $\vec{r}\cdot\vec{r}_1 4\pi r^2 = 4\pi r^3$ or, using the divergence theorem, for a sphere of radius r,

$$\int_s \vec{r}.\vec{r}_1 da = \int_\tau \nabla.\vec{r} d\tau = 4\pi r^3$$

<u>1-16</u> (1.8)

$\nabla.(f\vec{A}) = (\partial/\partial x)(fA_x) + (\partial/\partial y)(fA_y) + (\partial/\partial z)(fA_z)$

$= f_{\quad x \quad y \quad z \quad x \quad y \quad z}$

$= f\nabla.\vec{A} + \vec{A}.\nabla f$

<u>1-17</u> (1.8)

a) $f\vec{A} = r^2\vec{A}$, $(\partial/\partial x)(x^2+y^2+z^2) = 2x$, etc

$\nabla.(f\vec{A}) = (\partial/\partial x)(3xr^2) + (\partial/\partial y)(yr^2) + (\partial/\partial z)(2zr^2)$

$= 3r^2 + 3x\ 2x + r^2 + y\ 2y + 2r^2 + 2z\ 2z$

$= 6r^2 + 6x^2 + 2y^2 + 4z^2 = 12x^2 + 8y^2 + 10z^2 = 120$

b) $\nabla f = \nabla r^2 = (\partial/\partial x) r^2 \vec{i} + (\partial/\partial y) r^2 \vec{j} + (\partial/\partial z) r^2 \vec{k} = 2\vec{r}$

$$\nabla . \vec{A} = 3 + 1 + 2 = 6$$

$$f\nabla . \vec{A} + \vec{A} . \nabla f = 6r^2 + (3x\vec{i} + y\vec{j} + 2z\vec{k}) \cdot (2x\vec{i} + 2y\vec{j} + 2z\vec{k})$$

$$= 6r^2 + 6x^2 + 2y^2 + 4z^2 = 120$$

c) $\left[\nabla . (f\vec{A})\right] = \left[1/L\right]\left[LL^2\right] = \left[L^2\right]$

So $\nabla . (f\vec{A})$ is expressed in meters squared.

1-18 (1.7)

$$V = \int_{x=-R}^{x=+R} \int_{y=-(R^2-x^2)^{\frac{1}{2}}}^{y=+(R^2-x^2)^{\frac{1}{2}}} (H/2)(1-x/R)\,dy\,dx = \int_{-R}^{+R} (H/2)(1-x/R)\,2(R^2-x^2)^{\frac{1}{2}}dx$$

$$= H\int_{-R}^{+R} (R^2-x^2)^{\frac{1}{2}}dx - (H/R)\int_{-R}^{+R} (R^2-x^2)^{\frac{1}{2}}x\,dx = H(\pi/2)R^2 - 0 = H\pi R^2/2$$

1-19 (1.7)

Calculate the volume in the octant where x, y, z are all positive.

$$V/8 = \int_0^R \int_0^{(R^2-z^2)^{\frac{1}{2}}} \int_0^{(R^2-y^2-z^2)^{\frac{1}{2}}} dx\,dy\,dz = \int_0^R \int_0^{(R^2-z^2)^{\frac{1}{2}}} (R^2-y^2-z^2)^{\frac{1}{2}}dy\,dz$$

$$= \int_0^R (\pi/4)(R^2-z^2)\,dz = (\pi/4)R^3 - (\pi/4)(R^3/3) = (\pi/6)R^3$$

1-20 (1.9)

$$\int_\tau \nabla . \vec{A}\, d\tau = \int_S \vec{A} . \vec{da}$$

Now $\nabla . \vec{A} = df(x)/dx$, and $\vec{A} . \vec{da}$ is zero on the cylindrical surface. If the cross-section of the cylinder is B,

$$\int_a^b [df(x)/dx]\,B\,dx = \left[f(b) - f(a)\right]B$$

Thus

$$\int_a^b [df(x)/dx]\,dx = f(b) - f(a)$$

1-21 (1.10)

Set $F = K/r^2$. Then $PE = \int_r^\infty (K/r^2)dr = -K/r$

1-22 (1.10)

The work done by

curve is zero, even taking into account the curvature of the Earth. Then the gravitational field is conservative.

1-23 (1.12)

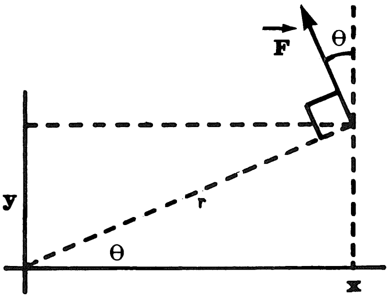
Since the field is conservative,

$$\underset{\text{over P}}{\int_a^b \vec{A}.d\vec{\ell}} + \underset{\text{over Q}}{\int_b^a \vec{A}.d\vec{\ell}} = 0 \Rightarrow \underset{\text{over P}}{\int_a^b \vec{A}.d\vec{\ell}} = \underset{\text{over Q}}{\int_a^b \vec{A}.d\vec{\ell}}$$

1-24 (1.12)

Since the value of the integral is independent of the path, the field defined by the

1-25 (1.12)



Since the force is azimuthal,

$$\oint \vec{F}.d\vec{\ell} = 2\pi rF \neq 0$$

and, from Stokes's theorem, $\nabla \times \vec{F} \neq 0$, so the force is non-conservative.

The curl is calculated as follows:

2 2 0.5

$F_x = -F\sin\theta = -Fy$

$F_y = F\cos\theta = Fx/r = Fx/(x^2+y$

where $F = K(x^2+y^2)^{0.2}$. So

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\dfrac{Ky}{(x^2+y^2)^{0.3}} & \dfrac{Kx}{(x^2+y^2)^{0.3}} & 0 \end{vmatrix}$$

$$= \left[\frac{K}{(x^2+y^2)^{0.3}} - \frac{0.3\ Kx}{(x^2+y^2\ 1.3}\,2x + \frac{K}{(x^2+y^2)^{0.3}} - \frac{0.3\ Ky}{(x^2+y^2)^{1.3}}\,2y\right]\vec{k}$$

$$= \frac{K}{(x^2+y^2)^{0.3}}\left[2 - \frac{0.3(2x^2+2y^2)}{(x^2+y^2)}\right]\vec{k} = \frac{1.4K}{r^{0.6}}\vec{k}$$

1-26 (1.11)

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xf(r) & yf(r) & zf(r) \end{vmatrix}$$

$$= \left[z(\partial f/\partial y) - y(\partial f/\partial z)\right]\vec{i} + \ldots$$

Now $\partial f/\partial y = (\partial f/\partial r)(\partial r/\partial y) = (\partial f/\partial r)(y/r), \partial f/\partial z = (\partial f/\partial r)(z/r)$

So $\nabla \times \vec{A} = \left[(zy/r)(\partial f/\partial r) - (yz/r)(\partial f/\partial r)\right] + \ldots = 0$

1-27 (1.11)

$$\nabla \times f\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ fA_x & fA_y & fA_z \end{vmatrix}$$

$$= \left[(\partial/\partial y)(fA_z) - (\partial/\partial z)(fA_y)\right]\vec{i} + \ldots$$

$$= \left\{ f\left[(\partial A_z/\partial y) - (\partial A_y/\partial z)\right] + A_z(\partial f/\partial y) - A_y(\partial f/\partial z)\right\}\vec{i} + \ldots$$

1-28 (1.11)

$$\nabla \cdot (\vec{A} \times \vec{D}) = (\partial/\partial x)(A_yD_z - A_zD_y)$$
$$+ (\partial/\partial y)(A_zD_x - A_xD_z)$$
$$+ (\partial/\partial z)(A_xD_y - A_yD_x)$$

$$\vec{D} \cdot (\nabla \times \vec{A}) = D_x(\partial A_z/\partial y - \partial A_y/\partial z) + D_y(\partial A_x/\partial z - \partial A_z/\partial x)$$
$$+ D_z(\partial A_y/\partial x - \partial A_x/\partial y)$$

$$-\vec{A} \cdot (\nabla \times \vec{D}) = -A_x(\partial D_z/\partial y - \partial D_y/\partial z) - A_y(\partial D_x/\partial z - \partial D_z/\partial x)$$
$$- A_z(\partial D_y/\partial x - \partial D_x/\partial y)$$

1-29 (1.11)

$$\nabla.\nabla \times \vec{A} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} \equiv 0$$

1-30 (1.12)

$$\oint_{c} \vec{E}.d\vec{\ell} = \int_{s} (\nabla \times \vec{E}).d\vec{a} = - \int_{s} (\partial\vec{B}/\partial t).d\vec{a} = 2 \times 10^{-3} \times 10^{-2} = 2 \times 10^{-5} V = 20\ \mu V$$

1-31 (1.13)

$$\nabla^2(\nabla f) = \vec{i}(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)(\partial f/\partial x) + \vec{j}\ldots$$
$$= \vec{i}(\partial/\partial x)(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2) f + \vec{j}\ldots$$
$$= \nabla(\nabla^2 f)$$

CHAPTER 2

2-1 (2.1) COULOMB'S LAW

a) $Ee = mg$, $E = mg/e = 9.1 \times 10^{-31} \times 9.8/1.6 \times 10^{-19} = 5.6 \times 10^{-11}$ V/m

b) $E = e/4\pi\epsilon_o r^2$, $r^2 = e/4\pi\epsilon_0 E$, $r = 5.1$ m

2-2 (2.1) SEPARATION OF PHOSPHATE FROM QUARTZ

Let x be the horizontal coordinate and y the vertical coordinate, downward

$x = (\frac{1}{2})(QE/m)t^2$, $y = (\frac{1}{2})gt^2$

$x/y = QE/mg = (Q/m)(E/g) = 10^{-5}(5 \times 10^{5}/9.8) \approx 0.5$

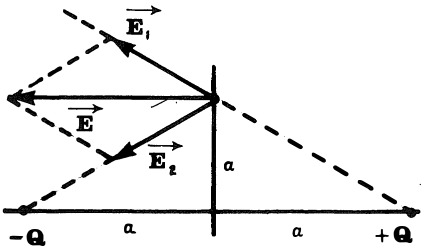
$y \approx 2x \approx 100$ mm

Reference: A.D. Moore, Electrostatics and its Applications, Wiley, 1973.

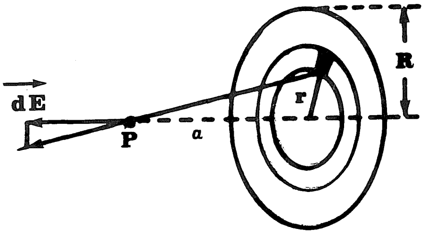
2-3 (2.3) ELECTRIC FIELD INTENSITY

$\vec{E} = \vec{E}_1 + \vec{E}_2$. By symmetry, the vertical components cancel and

$$\vec{E} = -2 \cos 45^o\ Q\vec{i}/4\pi\epsilon_o(a^2 + a^2)$$
$$= -\left[Q\Big/\left(2^{5/2}\pi\epsilon_o a^2\right)\right]\vec{i}$$



2-4 (2.4) ELECTRIC FIELD INTENSITY



The charge in the ring is $2\pi r dr\sigma$. Each point in the ring is at a distance $(a^2 + r^2)^{\frac{1}{2}}$ from P. By symmetry, E is along the axis.

$$dE = \frac{2\pi r dr\sigma}{4\pi\epsilon_o\left(a^2 + r^2\right)} \frac{a}{\left(a^2 + r^2\right)^{\frac{1}{2}}} = \sigma a r dr/2\epsilon_o\left(a^2 + r^2\right)^{3/2}$$

$$E = \frac{\sigma a}{2\epsilon_o}\int_0^R r dr/(a^2 + r^2)^{3/2} = -(\sigma a/2\epsilon_o)[1/(a^2 + r^2)^{\frac{1}{2}}]_0^R$$

$$= (\sigma a/2\epsilon_o)[1/(a^2 + R^2)^{\frac{1}{2}} - 1/a] \rightarrow \sigma/2\epsilon_o \text{ when } a << R$$

2-5 (2.5) CATHODE-RAY TUBE

Of course not. In approaching one plate an electron gains kinetic energy by losing potential energy, like a body falling in the gravitational field of the earth.

2-6 (2.5) CATHODE-RAY TUBE

Let V be the accelerating voltage and e the absolute value of the electronic charge. Then

$(1/2)mv^2 = eV, \quad v = (2eV/m)^{\frac{1}{2}}$.

If the distance traveled is D, the time of flight is $D(m/2eV)^{\frac{1}{2}}$.

During that time the electron falls by a distance

$$(1/2)gt^2 = 4.9\ D^2m/2eV = 4.9(0.2)^2\ 9.1 \times 10^{-31}/2 \times 1.6 \times 10^{-19} \times 5 \times 10^3$$
$$= 1.1 \times 10^{-16}\text{m}.$$

An atom has a diameter of the order of 10^{-10}m.

2-7 (2.5) MACROSCOPIC PARTICLE GUN

$$Q = 1.65 \times 4\pi \times 2 \times 8.85 \times 10^{-12} \times (10^{-12}/4)(1.5 \times 10^4/10^{-2}) = 1.38 \times 10^{-16}\text{C}$$

$$m = (4/3)\pi\ (10^{-18}/8) \times 1000 = 5.24 \times 10^{-16}\ \text{kg}$$

$$(1/2)mv^2 = 1.5 \times 10^4 \times 1.38 \times 10^{-16}, \quad v = 89\ \text{m/s}$$

Reference: A.D. Moore, p 59.

2-8 (2.5) ELECTROSTATIC SPRAYING

$QE/mg = (Q/m)(E/g) = E/g \geq 10^4/9.8 \approx 10^3$

Reference : A.D. Moore, pp 71, 250, 259, 262.

2-9 (2.5) THE RUTHERFORD EXPERIMENT

a) At the distance of closest approach, $Q_1Q_2/4\pi\epsilon_o r$ is equal to the kinetic energy :

$Q_1Q_2/4\pi\epsilon_o r = 7.68\times10^6\times1.6\times10^{-19}$

$r = 2\times79\times(1.6\times10^{-19})^2/4\pi\times8.85\times10^{-12}\times7.68\times1.6\times10^{-13} = 29.6$ fm

b) $Q_1Q_2/4\pi\epsilon_o r^2 = 7.68\times1.6\times10^{-13}/29.6\times10^{-15} = 41.5$ N

c) $a = (41.5/4\times1.7\times10^{-27})/9.8 = 6.23\times10^{26}$ g's

2-10 (2.5) ELECTROSTATIC SEED-SORTING

a) $2\times10^{-2} = (1/2)g\left[(t+0.01)^2 - t^2\right] \approx 5(0.02t+10^{-4})$, $t \approx 0.2$ s

The upper pea must have fallen through a distance of $gt^2/2$, or about 200 mm.

b) The average mass of one pea is $2000/100\times3600\times24 = 2.32\times10^{-4}$ kg. Thus $4\times10^{-2} = (1/2)(QE/m)t'^2$, where t' is the time interval during which a pea is deflected:

$t' = (2\times4\times10^{-2}\times2.32\times10^{-4}/1.5\times10^{-9}\times5\times10^5)^{\frac{1}{2}} = 0.157$ s.

The plates have a length $L = v_o t' + gt'^2/2$, with $v_o^2 = 2g\times0.2$, $v_o = 2$m/s

$L = 2\times0.157 + 4.9\times0.157^2 \approx 450$ mm.

Reference: Fluorescence-Activated Cell-Sorting, Scientific American, March 1976, p 108.

2-11 (2.5) CYLINDRICAL ELECTROSTATIC ANALYSER

$mv^2/R = QE = QV/a, \quad v = (QVR/ma)^{\frac{1}{2}}$

Reference: Jour. Phys. E, Sci. Instr. 10, 403 (1977).

2-12 (2.5) PARALLEL-PLATE ANALYSER

Reference: Rev. Sci. Instr. 42, 1423 (1971), Rev. Sci. Instr. 48, 454 (1977).

2-13 (2.5) CYLINDRICAL AND PARALLEL-PLATE ANALYSERS COMPARED

In the cylindrical analyser, $v = (QVR/ma)^{\frac{1}{2}}$.
So $(1/2)mv^2/Q = (R/2a)V$. For a given instrument, R and a are fixed and V is a measure of the given ratio.
In the parallel-plate analyser, from Prob. 2-12,
$QV_o/Q = (1/2)mv^2/Q = (a/2b)V$.
So V is a measure of the same ratio.

2-14 (2.5) ION THRUSTER

Consider a satellite of mass M and velocity V in a region where gravitational forces are negligible. The satellite ejects m'kg/sec backwards at a velocity v with respect to the satellite. The momentum of the system (M + total ejected mass) is constant. Then, with respect to a fixed reference frame, calling p the total momentum of the ejected fuel,
$(d/dt)(MV) + (dp/dt) = 0$, or $MdV/dt + VdM/dt + m'(V - v) = 0$,
$MdV/dt - m'V + m'V - m'v = 0$, $M(dV/dt) = m'v$.
It is this quantity that is called the thrust. Note that the thrust is not the force $(d/dt)(MV)$. In the last equation

a) $F = m'v$, $(1/2)m'v^2 = IV$, $m' = (I/ne)m$

$v^2 = 2IV/(I/ne)m$, $F = (2mV/ne)^{\frac{1}{2}}I$

b) $(2 \times 1.7 \times 10^{-27} \times 5 \times 10^4/1.6 \times 10^{-19})^{\frac{1}{2}}\ 0.1 = 3.26 \times 10^{-2}N$

c) $F = m'v$, $(1/2)m'v^2 = P$

$F = (2m'P)^{\frac{1}{2}} = 2\ P/v = 2P/(2IV/m')^{\frac{1}{2}} = (2m'/IV)^{\frac{1}{2}}P = (2m/neV)^{\frac{1}{2}}P$

d) $V = Q/4\pi\epsilon_o R$, $Q = 4\pi\epsilon_o RV = 4\pi \times 8.85 \times 10^{-12} \times 1 \times 5 \times 10^4 = 5.56 \times 10^{-6}V$

$t = Q/I = 10Q = 5.6 \times 10^{-5}s$

Reference: R.G. Jahn, Physics of Electric Propulsion.

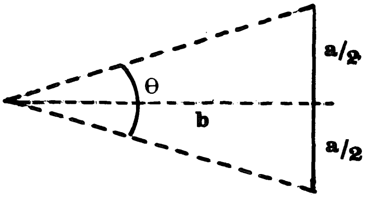
2-15 (2.5) COLLOID THRUSTER

Reference: The Electrical Propulsion of Space Vehicles, A.W. Bright and B. Makin, Contemporary Physics 14 (1973) p 25. See also Static Electrification 1975, The Institute of Physics, London, 1975, p 44.

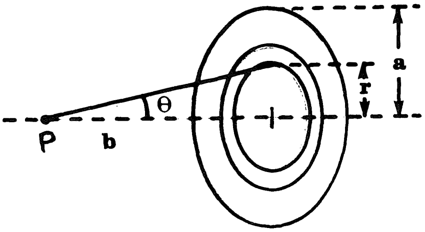
CHAPTER 3

3-1 (3.1) ANGLE SUBTENDED BY A LINE AT A POINT

$\theta = 2 \arctan (a/2b)$



3-2 (3.1) SOLID ANGLE SUBTENDED BY A DISK AT A POINT



The ring of radius r and width dr subtends at P a solid angle

$$d\Omega = [2\pi r dr/(b^2 + r^2)]\cos\theta$$

$$= 2\pi r dr b/(b^2 + r^2)^{3/2}$$

Thus

$$\Omega = 2\pi b \int_0^a r dr/(b^2 + r^2)^{3/2}$$

$$= 2\pi b \left[-(b^2 + r^2)^{-\frac{1}{2}} \right]_0^a$$

$$= 2\pi[1 - (1 + a^2/b^2)^{-\frac{1}{2}}]$$

If $b \gg a$, $\Omega \to 0$. If $b = 0$, $\Omega = 2\pi$. If $\Omega = \pi$, $a^2 = 3b^2$.

3-3 (3.2) GAUSS'S LAW

No. Gauss's law can only tell us that the net flux of $\vec{E}$ emitted by a dipole is zero, since the net charge is zero. For example, the average radial E is zero.

3-4 (3.2) SURFACE DENSITY OF ELECTRONS ON A CHARGED BODY

a) $\sigma = \epsilon_0 E = 8.85 \times 10^{-12} \times 3 \times 10^6 = 2.7 \times 10^{-5}$ C/m^2

b) Each atom occupies an area of about $(3 \times 10^{-10})^2$ meter^2. Thus the number of atoms per square meter is about 10^{19}.

c) The number of electrons per square meter is $2.7 \times 10^{-5}/1.6 \times 10^{-19}$, or 1.7×10^{14}. The number of free electrons per atom is $1.7 \times 10^{14}/10^{19}$ or 1.7×10^{-5}.

3-5 (3.2) THE ELECTRIC FIELD IN A NUCLEUS

$R = 1.25 \times 10^{-15}(127)^{1/3} = 6.28 \times 10^{-15}$m. At the center,

$V = (\rho/2\epsilon_o)R^2 = \left[Q^2/(4\pi R^3/3)\right](R^2/2\epsilon_o) = 3Ze/8\pi\epsilon_o R$

$= 3 \times 53 \times 1.6 \times 10^{-19}/8\pi \times 8.85 \times 10^{-12} \times 6.28 \times 10^{-15} = 1.8 \times 10^7$V

At the surface,

$V = (\rho/\epsilon_o)(R^2/2 - R^2/6) = \rho R^2/3\epsilon_o = V_{center}/1.5 = 1.2 \times 10^7$V,

$E = Q/4\pi\epsilon_o R^2 = 53 \times 1.6 \times 10^{-19}/4\pi \times 8.85 \times 10^{-12}(6.28 \times 10^{-15})^2$

$= 1.9 \times 10^{21}$V/m.

3-6 (3.2) THE SPACE DERIVATIVES OF E_x, E_y, E_z

From Gauss's law, $\partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z = \rho/\epsilon_o$.

Since $\nabla \times \vec{E} = 0$, $\partial E_z/\partial y = \partial E_y/\partial z$, $\partial E_x/\partial z = \partial E_z/\partial x$, $\partial E_y/\partial x = \partial E_x/\partial y$.

3-7 (3.2) PHYSICALLY IMPOSSIBLE FIELDS

We set $\vec{E} = E\vec{k}$

If $\rho = 0$, $\nabla\cdot\vec{E} = 0$ and $\partial E/\partial z = 0$.

Also, $\nabla \times \vec{E} = 0$ and $\partial E/\partial x = \partial E/\partial y = 0$.

So, if $\rho = 0$, $\vec{E}$ is uniform.

If $\rho \neq 0$, $\nabla \cdot \vec{E} = \rho/\epsilon_o$ and $\partial E/\partial z = \rho/\epsilon_o$.

Also, $\nabla \times \vec{E} = 0$ and $\partial E/\partial x = \partial E/\partial y = 0$, as before.

Then $\vec{E}$ is a function of z, but independent of x and y.

3-9 (3.4) ION BEAM

From Laplace's equation, $\partial^2V/\partial x^2 = -\rho/\epsilon_0$, $\partial V/\partial x = -(\rho/\epsilon_0)x + A$,

$V = -(\rho/2\epsilon_0)x^2 + Ax + B$.

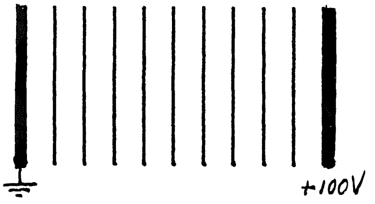
Since V = 0 at x = 0, B = 0. Also,

$V_0 = -(\rho/2\epsilon_0)a^2 + Aa$ and $A = V_0/a + \rho a/2\epsilon_0$.

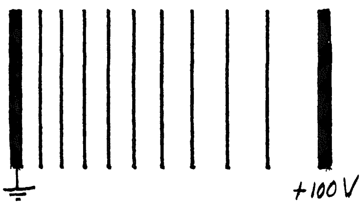
Finally, $V = (V_0/a + \rho a/2\epsilon_0)x - \rho x^2/2\epsilon_0$,

$E = -dV/dx = -(V_0/a + \rho a/2\epsilon_0) + \rho x/\epsilon_0$.

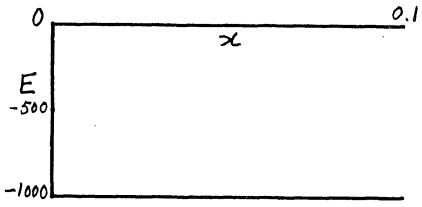
3-10 (3.4) A UNIFORM AND A NON UNIFORM FIELD



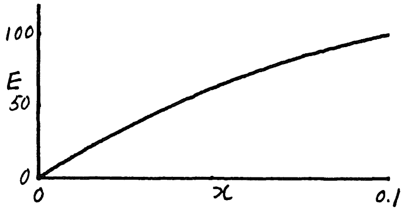
(a)



(b)



(c)



(d)

a) V = 1000 x, E = -1000. See Figs. a and c.

b) $\partial^2V/\partial x^2 = -10^4$, $\partial V/\partial x = -10^4x + A$, $V = -10^4x^2/2 + Ax + B$

Since V = 0 at x = 0, B = 0. Since V = 100 at x = 0.1, then A = 500.

$V = -10^4x^2/2 + 1{,}500x$. See Figs. b and d.

3-11 (3.4) VACUUM DIODE

a) $\partial^2V/\partial x^2 = (4V_o/9s^{4/3})x^{-2/3}$, $\partial V/\partial x = (4V_o/9s^{4/3})3x^{1/3} + A$

$V = (12\,V_o/9s^{4/3})(3/4)x^{4/3} + Ax = V_o(x/s)^{4/3} + Ax$

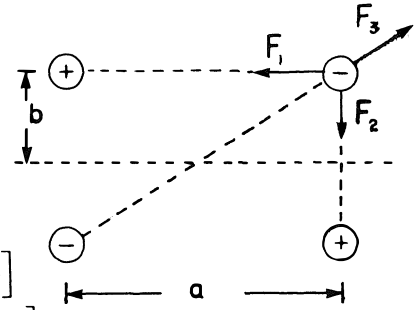
Since $V = V_o$ at $x = s$, then $A = 0$, $V = V_o(x/s)^{4/3}$

b) $J = \rho v$, $(1/2)mv^2 = eV_o$, $v = (2eV_o/m)^{\frac{1}{2}}$

$$J = -(4\epsilon_o V_o/9s^2)(2eV_o/m)^{\frac{1}{2}} = -(2^{5/2}\epsilon_o/9)(e/m)^{\frac{1}{2}}(V_o^{3/2}/s^2)$$

$$= -2.335 \times 10^{-6} V_o^{3/2}/s^2$$

3-12 (3.7) IMAGES



$F_1 = Q^2/4\pi\epsilon_o a^2$, $F_2 = Q^2/16\pi\epsilon_o b^2$

$F_3 = Q^2/4\pi\epsilon_o(a^2 + 4b^2)$

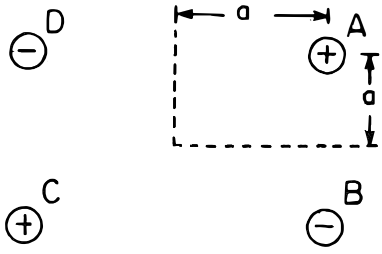
$F_{3x} = Q^2a/4\pi\epsilon_o(a^2 + 4b^2)^{3/2}$

$F_{3y} = 2Q^2b/4\pi\epsilon_o(a^2 + 4b^2)$

$F_x = (Q^2/4\pi\epsilon_o)\left[a/(a^2 + 4b^2)^{3/2} - 1/a^2\right]$

$F_y = (Q^2/4\pi\epsilon_o)\left[2b/(a^2 + 4b^2)^{3/2} - 1/4b^2\right]$

3-13 (3.7) IMAGES



$\vec{E}_A = -(Q/4\pi\epsilon_o a^2)\vec{j}$

$\vec{E}_B = -(Q/4\pi\epsilon_o a^2)\vec{j}$

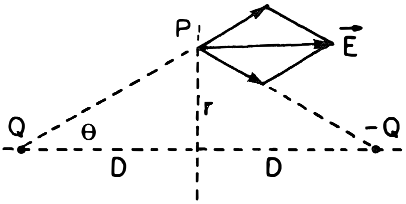
$\vec{E}_C = \left[Q/4\pi\epsilon_o(a^2 + 4a^2)^{3/2}\right](2a\vec{i} + a\vec{j})$

$\vec{E}_D = \left[Q/4\pi\epsilon_o(a^2 + 4a^2)^{3/2}\right](-2a\vec{i} + a\vec{j})$

$\vec{E}_{tot} = (Q/4\pi\epsilon_o a^2)(-2 + 2/5\sqrt{5})\vec{j}$

$= -(1.8211Q/4\pi\epsilon_o a^2)\vec{j}$

3-14 (3.7) IMAGES



a) At some point P on the conducting plane,

$$E = 2\left[Q/4\pi\epsilon_o(D^2 + r^2)\right]\cos\theta$$

$$= 2QD/4\pi\epsilon_o(D^2 + r^2)^{3/2}$$

$$\sigma = -\epsilon_o E = -QD/2\pi(D^2 + r^2)^{3/2}$$

$$b)\ -\int_0^\infty 2\pi r dr QD/2\pi(D^2 + r^2)^{3/2} = -QD\int_0^\infty r dr/(D^2 + r^2)^{3/2} = -Q$$

CHAPTER 4

4-1 (4.1) THE PERMITTIVITY OF FREE SPACE

From Coulomb's law,

$[\epsilon_o] = [Q^2/FL^2] = [Q^2/(FL)L] = [Q^2/(Q^2/C)L] = [C/L]$,

where the brackets indicate that we are concerned only with the dimensions, and where F, L, C, stand for Force, Length, and Capacitance. Note that FL is an energy, like Q^2/C.

4-2 (4.1) THE EARTH'S ELECTRIC FIELD

a) $C = 4\pi\epsilon_o R = 7.1 \times 10^{-4} F \approx 700 \mu F$

b) $Q = 4\pi R^2\sigma = 4\pi R^2\epsilon_o E = (6.4 \times 10^6)^2 \times 100/9 \times 10^9 = 4.5 \times 10^5 C$

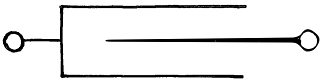
c) $V = 4\pi R^2\epsilon_o E/4\pi\epsilon_o R = ER = 6.4 \times 10^8 V$

Reference: Richard Feynman, Lectures on Physics, 2, Ch 9, Addison-Wesley.

4-3 (4.2) PARALLEL-PLATE CAPACITOR

If there are 3 plates,

$C = 8.85 \times 10^{-12}(2A/t)$



With four plates,

$C = 8.85 \times 10^{-12}(3A/t)$, etc.

For N plates,

$C = 8.85 \times 10^{-12}(N-1)A/t\ F$

$= 8.85\ (N-1)A/t)\ pF$

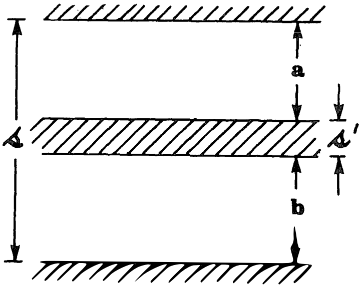
4-4 (4.2) PARALLEL-PLATE CAPACITOR

The plate separation might be 1 mm. Then $10^{-12} = 8.85 \times 10^{-12}A/10^{-3}$, $A \approx 10^{-4}$, or one square centimeter.

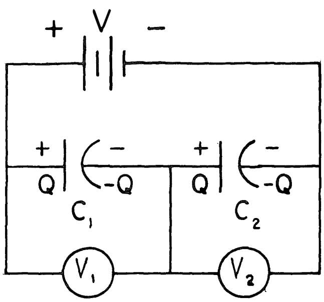
4-5 (4.2) PARALLEL-PLATE CAPACITOR

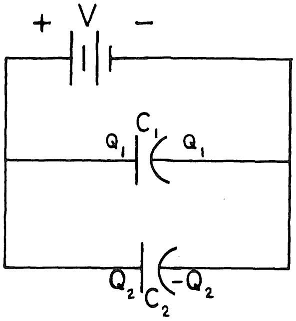
$C' = C_aC_b/(C_a+C_b) = \epsilon_o S/(a+b) = \epsilon_o S/(s-s')$

The capacitance is larger, but it is independent of the position of the conducting plate.



4-8 (4.3) ELECTROSTATIC ENERGY

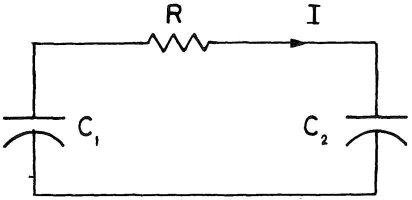




a) $W_1/W_2 = (QV_1/2)/(QV_2/2) = V_1/V_2 = (Q/C_1)/(Q/C_2) = C_2/C_1$

b) $W_1/W_2 = (Q_1V/2)/(Q_2V/2) = Q_1/Q_2 = C_1V/C_2V = C_1/C_2$

4-9 (4.3) ELECTROSTATIC ENERGY



a) The energy that is dissipated is

$$\left(\frac{Q_1^2}{2C_1}+\frac{Q_2^2}{2C_2}\right) - \frac{\left(Q_1+Q_2\right)^2}{2(C_1+C_2)}.$$

$$= \frac{C_2(C_1+C_2)Q_1^2+C_2(C_1+C_2)Q_2^2-C_1C_2(Q_1+Q_2)^2}{2C_1C_2(C_1+C_2)}$$

$$= (C_1Q_2-C_2Q_1)^2/2C_1C_2(C_1+C_2)$$

b) The energy is dissipated by Joule heating in the resistance R of the wires. Let Q_{10} and Q_{20} be the charges at $t = 0$, Q_1 and Q_2 the charges at t. C_1 discharges into C_2. Then

$Q_1/C_1 - Q_2/C_2 = IR$, $Q_1+Q_2 = Q_{10}+Q_{20} = Q$, $I = dQ_2/dt$,

$$(Q-Q_2)/C_1 - Q_2/C_2 = RdQ_2/dt, \quad RdQ_2/dt + Q_2(1/C_1+1/C_2) = Q/C_1$$

$$Q_2 = \frac{Q/C_1}{1/C_1+1/C_2} + A \exp[-(1/C_1+1/C_2)t/R]$$

Since $Q_2 = Q_{20}$ at $t = 0$, $A = Q_{20} - \dfrac{Q/C_1}{1/C_1+1/C_2}$

$$dQ_2/dt = \left[Q_{20} - \frac{Q/C_1}{1/C_1+1/C_2}\right] [-(1/C_1+1/C_2)/R] \exp[-(1/C_1+1/C_2)t/R]$$

$$= [-Q_{20}(1/C_1+1/C_2)/R + Q/C_1R] \exp[-(1/C_1+1/C_2)t/R]$$

$$= [-Q_{20}/C_2R + Q_{10}/C_1R] \exp[-(1/C_1+1/C_2)t/R]$$

$$W = \int_0^\infty (dQ_2/dt)^2 R = \frac{[Q_{10}/C_1R - Q_{20}/C_2R]^2}{-2(1/C_1+1/C_2)/R} R(0-1)$$

$$= \frac{(Q_{10}/C_1 - Q_{20}/C_2)^2}{2(1/C_1+1/C_2)} = \frac{(Q_{10}C_2 - Q_{20}C_1)^2}{2C_1C_2(C_1+C_2)}$$

This is the result found under a, except that the initial charges are now called Q_{10} and Q_{20}, instead of Q_1 and Q_2.

4-10 (4.3) PROTON BOMB

a) $W = (1/2)\displaystyle\int_0^R \rho V d\tau = (1/2)\int_0^R (\rho^2/\epsilon_0)(R^2/2 - r^2/6)4\pi r^2 dr$

$$= (4\pi/15\epsilon_0)\rho^2 R^5 = 3Q^2/(20\pi\epsilon_0 R)$$

b) Use the above result, replacing $1/4\pi\epsilon_0$ by G :

$W_G = 3GM^2/5R$

c) $3 \times 6.67 \times 10^{-11} \times (7.33 \times 10^{22})^2/5 \times 1.74 \times 10^6 = 1.24 \times 10^{29}$J.

d) $\rho = (1000/1.7 \times 10^{-27})1.6 \times 10^{-19} = 9.6 \times 10^{10}$ C/m^3

If R is the radius of the sphere of protons, $4\pi\rho^2R^5/15\epsilon_0 = 1.24 \times 10^{29}$,

$R = [15\epsilon_0 \times 1.24 \times 10^{29}/4\pi(9.6 \times 10^{10})^2]^{1/5} = 0.17$ m

4-11 (4.5) ELECTROSTATIC MOTOR

Reference: A.D. Moore, Electrostatics and its Applications.

4-12 (4.5) ELECTROSTATIC PRESSURE

a) $V = Q/4\pi\epsilon_0 R$, $E = Q/4\pi\epsilon_0 R^2$, $V = ER = 3\times 10^6/0.05 = 1.5\times 10^5$V

b) The pressure is $\sigma^2/2\epsilon_0 = (Q/4\pi R^2)^2/2\epsilon_0 = (Q/4\pi\epsilon_0 R)^2(\epsilon_0{}^2/R^2)/2\epsilon_0$

$= (1.5\times 10^5)^2\epsilon_0/2R^2 \approx 40$Pa $\approx 4\times 10^{-4}$ atmosphere.

4-13 (4.5) PARALLEL-PLATE CAPACITOR

Let each plate have an area S. Then the capacitance changes by

$dC = d(\epsilon_0 S/s) = -(\epsilon_0 S/s^2)ds.$

Let ds be positive. The capacitance decreases and a charge

$dQ = V|dC| = (\epsilon_0 S/s^2)Vds$ returns to the battery. Thus

$dW_B = -(\epsilon_0 S/s^2)V^2 ds = -\epsilon_0 E^2 Sds.$

The energy stored in the field increases by

$dW_e = d(\epsilon_0 E^2 sS/2) = d(\epsilon_0 V^2 S/2s) = -(\epsilon_0 V^2 S/2)ds/s^2 = -\epsilon_0 E^2 Sds/2.$

The mechanical work done <u>on</u> the system is

$Fds = S(\epsilon_0 E^2/2)ds = \epsilon_0 E^2 S/2.$

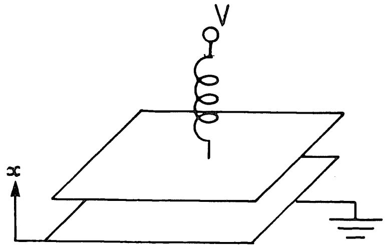
4-14 (4.5) PARALLEL-PLATE CAPACITOR

See the preceding problem. Here, $dW_B = 0$ and E is constant.

$dW_e = d(\epsilon_0 E^2 sS/2) = (\epsilon_0 E^2 S/2)ds$

$Fds = (\epsilon_0 E^2 S/2)ds$, $F = \epsilon_0 E^2 S/2$

4-15 (4.5) OSCILLATING PARALLEL-PLATE CAPACITOR



a) The energy stored in the capacitor is

$QV/2 = CV^2/2 = (\epsilon_0 A/x)V^2/2.$

Let $x > x_0$. Then the total potential energy is

$W = mg(x-x_0) + k(x-x_0)^2/2$

$+ \epsilon_0 A(1/x-1/x_0)V^2/2 + V\Delta Q,$

where ΔQ is the charge fed into the battery because of the decrease in capacitance:

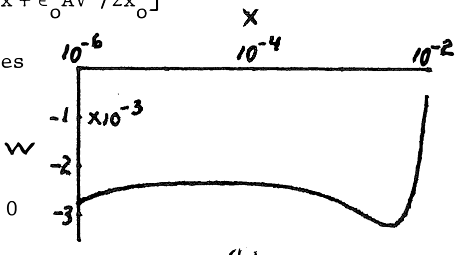
$V\Delta Q = -V^2\Delta C = -V^2\epsilon_0 A(1/x-1/x_0).$

The battery gains energy if ΔC is negative. Thus

$W = mg(x-x_0) + k(x-x_0)^2/2 - \epsilon_0 A(1/x-1/x_0)V^2/2$

$= (x-x_0)\left[mg + (x-x_0)k/2 + \epsilon_0 AV^2/2xx_0\right]$

$= [(x-x_o)/x][kx^2/2 + (mg-kx_o/2)x + \epsilon_o AV^2/2x_o]$



(b)

b) There are three downward forces and, at equilibrium,

$mg + k(x-x_o) + \epsilon_o AV^2/2x^2 = 0.$

Also, at equilibrium,

$dW/dx = mg + k(x-x_o) + \epsilon_o AV^2/2x^2 = 0$

c) The relation $F = -(dW/dx)_{eq}$ comes from the conservation of energy for a small displacement near equilibrium. In setting $dW/dx = K(x-x_{eq})$, we assume that the W(x) curve approximates a parabola $W = (K/2)(x-x_{eq})^2$ in the region near the point of stable equilibrium. Thus

$K = (d^2W/dx^2)_{eq} = k - \epsilon_o AV^2/x^3_{eq}$,

$\omega = \left[\dfrac{k - \epsilon_o AV^2/x^3_{eq}}{m}\right]^{\frac{1}{2}}$, f = 6.16 Hz.

4-16 (4.5) HIGH-VOLTAGE GENERATOR

a) The charge density on the plates, and hence E, remain constant when the plates are separated. Then the increase in energy is $\epsilon_o E^2 Ss(n-1)/2$. The mechanical work done is the force, $\epsilon_o E^2 S/2$, multiplied by s(n-1).

b) Reference: A.D. Moore, Electrostatics and its Applications, Chapter 8.

4-17 (4.5) INK-JET PRINTER

a) $\lambda = C'V = 2\pi\epsilon_o V/\ln(R_2/R_1)$

b) $(4/3)\pi(2R_1)^3/\pi R_1^2 = 32\ R_1/3$

c) $Q = 32\ R_1\lambda/3 = 64\pi\epsilon_o VR_1/3\ln(R_2/R_1)$

d) $m = 1000\tau$, $Q/m = Q/1000\tau = 64\pi\epsilon_o VR_1/3000\ \ln(R_2/R_1)(4/3)\pi(2R_1)^3$

$= \epsilon_o V/500\ R_1^2\ \ln(R_2/R_1)$

$Q/m = 8.85 \times 10^{-12} \times 100/500(2\times 10^{-5})^2 \ln(5\times 10^{-3}/2\times 10^{-5}) = 8.0\text{x}10^{-4}$C/kg

e) $v = 10^5 \times 10^{-4} = 10$ m/s

f) A droplet remains in the deflecting field during 4×10^{-3}s. During that time it is subjected to a transverse force QE and its acceleration is QE/m, or $8 \times 10^{-4} \times 10^5$, or 80 m/s^2. The transverse deflection is

$at^2/2 = 80(4 \times 10^{-3})^2/2 = 0.64$ mm

The transverse velocity at the far end of the deflecting plates is

$at = 80 \times 4 \times 10^{-3} = 0.32$ m/s

Reference: Special issue of the IBM Journal of Research and Development, January 1977.

CHAPTER 5

5-1 (5.1) CONDUCTION IN A UNIFORM MEDIUM

$\sigma = \sigma_o + ax/s$, $E = J/\sigma = J/(\sigma_o + ax/s)$

5-2 (5.2) RESISTIVE FILM

Let the film have an area a^2 and a thickness t. Then

$R = a/\sigma at = 1/\sigma t$.

5-3 (5.2) RESISTOJET

The thrust is m'v, where m' is the mass ejected per second, and v is the exhaust velocity. See the solution of Prob. 2-14. Then

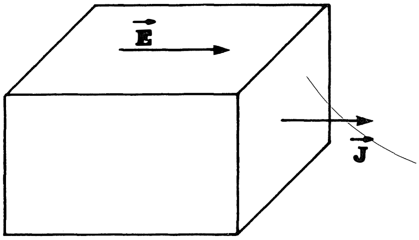
$m'v^2/2 = 3000$, $v = (6000/m')^{\frac{1}{2}}$, $m'v = (6000\ m')^{\frac{1}{2}} = 1.9$ N.

Reference: Robert J. Jahn, Physics of Electric Propulsion, p 103.

5-4 (5.2) JOULE LOSSES

$V^2/R = P$, $V^2 = RP = 10^5 \times 0.25$,

$V = 158$ V



5-5 (5.4) VOLTAGE DIVIDER

The current flowing through R_1 and R_2 is I.
$V_i = I(R_1 + R_2)$, $V_o = IR_2$, $V_o/V_i = R_2/(R_1 + R_2)$

5-6 (5.4) POTENTIOMETER

See Prob. 5-5.

5-7 SIMPLE CIRCUIT

$V' = V - 2V[R_1/(R_1 + R_2)] = (R_2 - R_1)V/(R_2 + R_1)$

5-9 AMPLIFIER

a) R_1 and R_2 carry the same current

$I = (V_i - V_{iA})/R_1 = (V_{iA} - V_o)/R_2$,

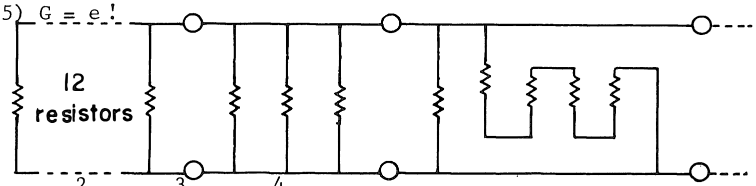
$(V_i + V_o/A)/R_1 = V_o(-1/A - 1)/R_2$,

$V_o/V_i = -R_2/[R_1 + (R_1 + R_2)/A] = -(R_2/R_1)/[1 + 1/A + (R_2/R_1)/A]$.

$V_o/V_i \approx -R_2/R_1$ if $A >> 1$ and if $R_2/R_1 << A$. The gain R_2/R_1 must therefore be much less than A.

b) $(R_1 + R_2)/A \leq R_1/1000$, $3000/A \leq 1$, $A \geq 3000$

5-10 (5.5) G = e !



$\exp x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \ldots$

$e = 1 + 1 + 1/2 + 1/6 + 1/24 + \ldots$

5-11 (5.5) TETRAHEDRON

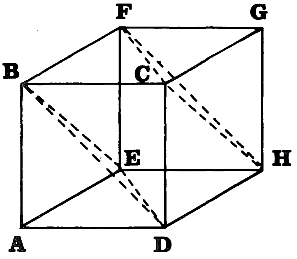
a) By symmetry, the currents through ACB and ADB are equal. The potential at C and D is half-way between the potentials at A and B.

b) $2R/2 = R$, in parallel with the R between A and B. The resistance between nodes A and B is R.

5-12 (5.5) CUBE

a) By symmetry points BED are at the same potential. Points FCH are at another potential.

b) The resistance from A to BED is R/3. That from BED to FCH is R/6. That from FCH to G is R/3. The resistance is 5R/6.



5-13 (5.5) CUBE

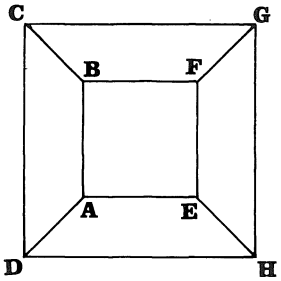
a) Points FBDH. Branches FD and DH can be either removed or short-circuited.

b) Remove those branches. Around the inner square, we have a resistance 2R/2 = R. Around the outer square, the resistance is 3R. Thus we have R and 3R in parallel and the resistance is 3R/4.

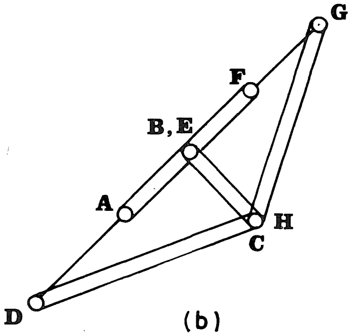
5-14 (5.5) CUBE

Distort the cube as in Fig. a. Then, by symmetry, B and E are at the same potential and can be shorted. Similarly, C and H can be shorted. Now redraw the figure as in b and c. The resistance to the right of the dotted line is 0.4 R and

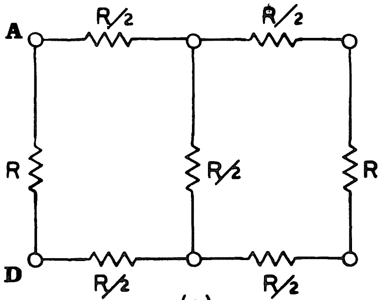
$R_{AD} = R(1.4R)/(R+1.4R) = 1.4R/2.4$
$= 0.583R$



(a)



(b)



(c)

5-15 (5.7) LINE FAULT LOCATION

Let the length of the line be ℓ and let a be the resistance of one meter of wire. Then

$2ax + R_s = 550/3.78 = 145.5, \quad R_s = 145.5 - 2ax,$

$2ax + R_s 2a(\ell - x)/[R_s + 2a(\ell - x)] = 550/7.2 = 76.39,$

$ax[R_s + 2a(\ell - x)] + R_s a(\ell - x) = 38.19[R_s + 2a(\ell - x)],$

$axR_s + 2a^2x\ell - 2a^2x^2 + R_s a(\ell - x) = 38.19R_s + 76.39a\ell - 76.39ax.$

Canceling the axR_s terms and substituting the value of R_s in the first equation,

$2a^2x\ell - 2a^2x^2 + (145.5 - 2ax)a\ell = 38.19(145.5 - 2ax) + 76.39a\ell - 76.39ax,$

$-2a^2x^2 + 152.8ax + 69.11a\ell - 5.557 = 0.$

Now $a = 1/5.8 \times 10^7 \pi(1.5 \times 10^{-3})^2 = 2.439 \times 10^{-3}.$

Solving, x = 7.818 kilometers.

5-16 (5.7) UNIFORM RESISTIVE NET

a) From Kirchoff's voltage law, the sum of the currents flowing into 0 is zero. Thus

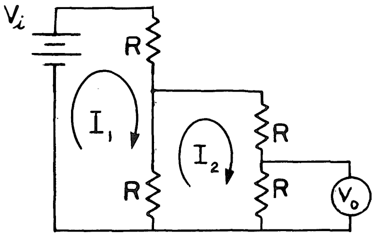
$(V_A - V_0)/R + (V_B - V_0)/R + (V_C - V_0)/R + (V_D - V_0)/R = 0,$

$V_0 = (V_A + V_B + V_C + V_D)/4.$

b) For a three-dimensional circuit we have 6 resistors connected to 0 and

$V_0 = (V_A + V_B + V_C + V_D + V_E + V_F)/6.$

5-17 (5.9) POTENTIAL DIVIDER



For the left hand mesh,

$V_i - I_1R - (I_1 - I_2)R = 0.$

For the middle mesh,

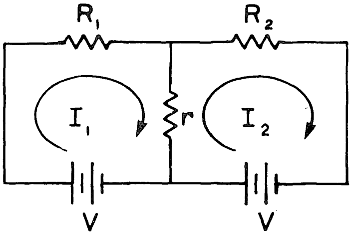
$(I_2 - I_1)R + I_2R + I_2R = 0.$

Then $I_2 = V_i/5R$, $V_o = (V/5R)R = V/5.$

5-18 (5.9) SIMPLE CIRCUIT WITH TWO SOURCES

$(R_1+r)I, - rI_2 = V, - rI_1 + (R_2+r)I_2 = V$

$I = I_1-I_2 = (R_2-R_1)V/[R_1R_2+r(R_1+R_2)]$



5-19 (5.10) DELTA-STAR TRANSFORMATIONS

Equating the voltages,

$$V_A-V_B = R_c(I_D-I_C) = R_A(I_B-I_C) + R_B(I_A-I_C), \quad (1)$$

$$V_B-V_C = R_a(I_D-I_A) = R_B(I_C-I_A) + R_C(I_B-I_A), \quad (2)$$

$$V_C-V_A = R_b(I_D-I_B) = R_C(I_A-I_B) + R_A(I_C-I_B). \quad (3)$$

Rewriting,

$$-I_AR_B - I_BR_A + I_C(R_A+R_B-R_c) + I_DR_c = 0, \quad (4)$$

$$-I_A(R_B+R_C-R_a) - I_BR_C - I_CR_B + I_DR_a = 0, \quad (5)$$

$$-I_AR_C + I_B(R_C+R_A-R_b) - I_CR_A + I_DR_b = 0. \quad (6)$$

Eliminating I_D from Eqs. 4 and 5,

$$-I_AR_B-I_BR_A + I_C(R_A+R_B-R_C) + (R_c/R_a)[-I_A(R_B+R_C-R_a) + I_BR_C + I_CR_B] = 0 \quad (7)$$

$$I_A[-R_B - (R_c/R_a)(R_B+R_C-R_a)] + I_B[-R_A + (R_c/R_a)R_C]$$

$$+ I_C[R_A+R_B-R_c + (R_c/R_a)R_B] = 0 \quad (9)$$

Thus $R_B + (R_c/R_a)(R_B+R_C) - R_c = 0, \; R_A/R_C = R_c/R_a$ (10)

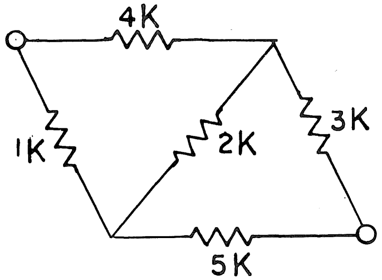
$$R_B + (R_c/R_a)R_B + R_A - R_c = 0 \quad (11)$$

Only two of these equations are independent. Combining the first two,

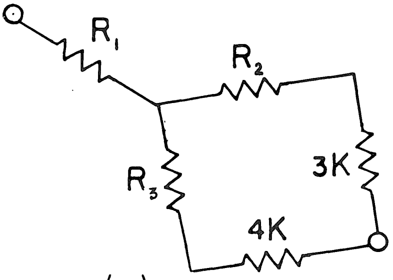
$$R_B + (R_A/R_C)R_B + R_A = R_c, \; R_c = (R_BR_C + R_CR_A + R_AR_B)/R_C \quad (12)$$

Or, setting $R = 1/G$, $G_c = G_AG_B/(G_A+G_B+G_C)$ (13)

5-20 (5.10) DELTA-STAR TRANSFORMATIONS



(a)



(b)

Redraw the circuit as in Fig. a and transform the left hand delta into a star, as in Fig. b, with

$R_1 = 4000 \times 1000/7000 = (4/7)1000\Omega$, $R_2 = (8/7)1000\Omega$, $R_3 = (2/7)1000\Omega$,

$R = 2.89\ \text{k}\Omega$.

5-21 (5.12) OUTPUT RESISTANCE OF A BRIDGE CIRCUIT

The output resistance is the resistance one would measure at the terminals of the voltmeter if the source were replaced by a short-circuit. This is $R/2 + R/2 = R$.

5-22 (5.12) INTERNAL RESISTANCE OF AN AUTOMOBILE BATTERY

The headlights, tail lights, etc draw about 15 A. Hence the internal resistance of the battery is about $(1/15)\Omega$. This is much too large, because a cranking motor draws, say 200 A. A normal automobile battery has an internal resistance of the order of $10^{-2}\Omega$.

Reference: Standard Handbook for Electrical Engineers, Sections 21 and 24.

5-23 (5.14) DISCHARGING A CAPACITOR THROUGH A RESISTOR

From Kirchoff's voltage law, $Q/C = RdQ/dt$. Thus

$RdQ/dt - Q/C = 0$, $Q = Q_o\exp(-t/RC)$, $V = V_o\exp(-t/RC) = 100\exp(-t)$

5-24 (5.14) RAMP GENERATOR

$V_o = Q/C = It/C$

5-25 (5.14) CHARGING A CAPACITOR THROUGH A RESISTOR

The energy supplied by the source is

$$W_s = \int_0^\infty VI dt = V\int_0^\infty (dQ/dt)dt = V\int_0^{CV} dQ = CV^2.$$

The energy stored in the capacitor for t → ∞ is $CV^2/2$.

The energy dissipated in the resistor is

$$W_R = \int_0^\infty I^2 R dt = R\int_0^\infty [(V/R)\exp(-t/RC)]^2 dt = CV^2/2$$

5-26 (5.14) RC TRANSIENT

a) $I = V/R_2 + CdV/dt$, $V_s = IR_1 + V = (V/R_2 + CdV/dt)R_1 + V$

$R_1C\ dV/dt + (1+R_1/R_2)V = V_s$

$$V = V_s/(1+R_1/R_2) + A\exp[-(1+R_1/R_2)t/R_1C]$$

Since V = 0 at t = 0,

$$V = [V_s/(1+R_1/R_2)]\{1-\exp[-(1+R_1/R_2)t/R_1C]\}$$

$$= [R_2/(R_1+R_2)]\{1-\exp[-(R_1+R_2)t/R_1R_2C]\}V_s$$

The time constant is $R_1R_2C/(R_1+R_2)$, or $C/(1/R_1+1/R_2)$.

For t → ∞, $V = R_2V_s/(R_1+R_2)$.

b) Now, at t = 0, $V = R_2V_s/(R_1+R_2)$.

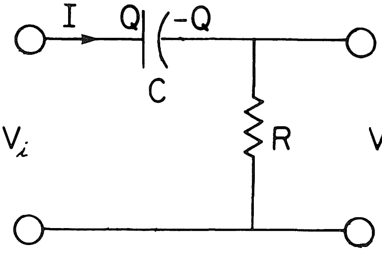
The capacitor discharges through R_2 and

$V = [R_2V_s/(R_1+R_2)]\exp[-t/R_2C]$.

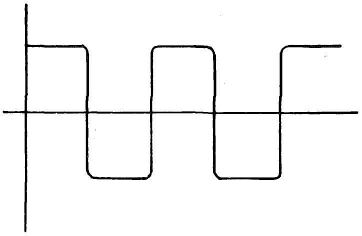
5-27 (5.14) RC DIFFERENTIATING CIRCUIT

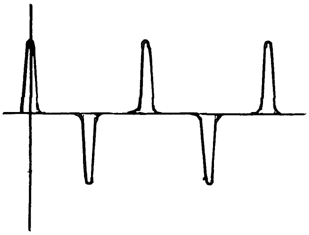
$V_i = Q/C + RI \approx Q/C$

$V_o = RI = RdQ/dt \approx RCdV_i/dt$



5-28 DIFFERENTIATING A SQUARE WAVE





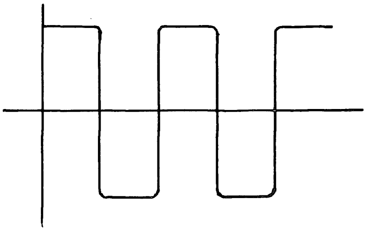
5-30 (5.14) RC INTEGRATING CIRCUIT

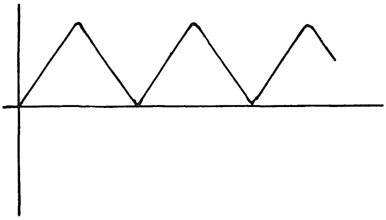
The current flowing into the capacitor is I.

$V_i = RI + Q/C = RdQ/dt + Q/C \approx RdQ/dt,$

$$V_o = Q/C = (1/RC) \int_0^t V_i dt.$$

5-31 (5.14) INTEGRATING CIRCUIT





5-32 (5.14) INTEGRATING CIRCUIT

$(V_i - V_{iA})/R = C(d/dt)(V_{iA} - V_o)$

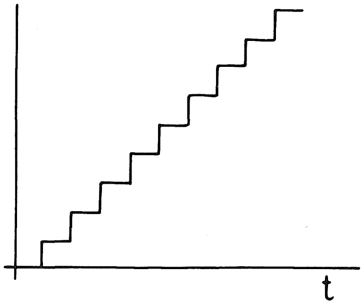
$V_i + V_o/A = -RC(1+1/A)dV_o/dt,$

$V_i = -RC(1+1/A)dV_o/dt - V_o/A$

$\quad = -RCdV_o/dt$ if $A >> 1$ and if $V_o/A << RCdV_o/dt$.

5-33 (5.14) PULSE-COUNTING CIRCUIT

a) During a pulse, the voltage across C_1 is approximately equal to V_p and $Q \approx C_1V_p$. After the first pulse, the voltage across C_2 is C_1V_p/C_2. The process repeats itself. The voltage across C_2 increases by C_1V_p/C_2 at each pulse.



b) $V \approx (C_1V_p/C_2)ft$.

CHAPTER 6

6-1 (6.1) THE DIPOLE MOMENT p

a) $p = P/N = 10^{-7}/[6.02 \times 10^{23} \times (3.5/12)10^{6}] = 5.7 \times 10^{-37}\,\mathrm{Cm}$

b) $s = p/Q = 5.7 \times 10^{-37}/6 \times 1.6 \times 10^{-19} = 5.9 \times 10^{-19}\ \mathrm{m}$

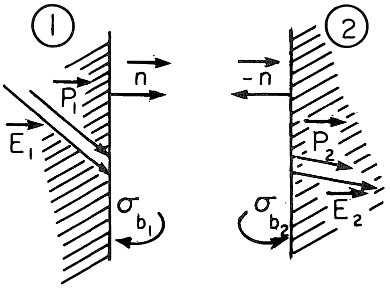
The diameter of an atom is of the order of 10^{-10} m.

6-2 (6.2) THE VOLUME AND SURFACE BOUND CHARGE DENSITIES

$$\int_\tau -\nabla \cdot \vec{P}d\tau + \int_S \sigma_b da = -\int_S \vec{P}.\vec{da} + \int_S \sigma_b da = \int_S (-\sigma_b + \sigma_b)da = 0$$

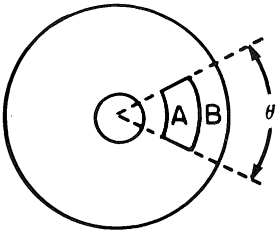
6-3 (6.2) BOUND CHARGE DENSITY AT AN INTERFACE

In the figure, we have shown the two media separated, for clarity. On the face of 1, $\sigma_{b1} = \vec{P}_1 \cdot \vec{n}$. On 2, $\sigma_{b2} = \vec{P}_2 \cdot (-\vec{n}) = -\vec{P}_2 \cdot \vec{n}$. Thus $\sigma_{net} = (\vec{P}_1 - \vec{P}_2) \cdot \vec{n}$



6-4 (6.4) COAXIAL LINE

Consider a volume τ of dielectric having the shape shown in the figure.



$$\int_\tau \nabla\cdot\vec{E}d\tau = \int_S \vec{E}\cdot d\vec{a}$$

where S is the surface bounding τ. The surfaces A and B are the only ones where $\vec{E}\cdot d\vec{a}$ is not zero. Then, if their radii are r_A and r_B,

$$\int_S \vec{E}\cdot d\vec{a} = -(\lambda/2\pi\epsilon_o r_A)r_A\theta L + (\lambda/2\pi\epsilon_o r_B)r_B\theta L = 0$$

So $\int_\tau \nabla\cdot\vec{E}d\tau = 0$ and, since r_A, r_B and θ are arbitrary, $\nabla\cdot\vec{E} = 0$.

6-5 (6.7) COAXIAL LINE

a) Near the inner conductor,

$E_1 = \lambda/2\pi\epsilon_r\epsilon_o R_1$

If λ is the charge per meter and C' the capacitance per meter,

$\lambda = C'V = [2\pi\epsilon_r\epsilon_o/\ell n(R_2/R_1)]V$

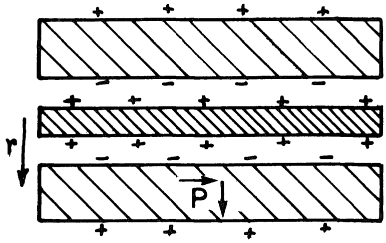
Thus $E_1 = V/R_1\ell n(R_2/R_1)$

$5\times10^6 = 500/R_1\ell n(5\times10^{-3}/R_1)$, $R_1 = 1.772\times10^{-5}$ m

b) One should use No 34 wire.

c) $C' = 2\pi\times2.1\times8.85\times10^{-12}/\ell n(5/0.16) = 33.93$ pF/m

6-7 (6.8) CHARGED WIRE EMBEDDED IN DIELECTRIC : THE FREE AND BOUND CHARGES



a) Inside the dielectric,

$D = \lambda/2\pi r$, $E = \lambda/2\pi\epsilon_r\epsilon_o r$

Then $P = D - \epsilon_o E = (\lambda/2\pi r)(1-1/\epsilon_r)$

On the inner surface of the dielectric,

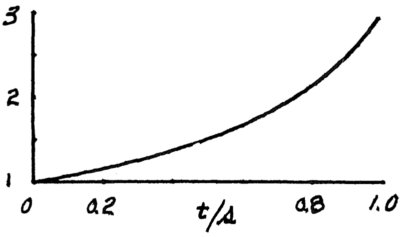
$\sigma_i = -P_i = -(\lambda/2\pi a)(1-1/\epsilon_r)$, $Q'_{bi} = -\lambda(1-1/\epsilon_r)$

On the outer surface,

$\sigma_o = P_o = (\lambda/2\pi b)(1-1/\epsilon_r)$, $Q_{bo} = \lambda(1-1/\epsilon_r)$

b) $\lambda + Q'_{bi} = \lambda - \lambda(1-1/\epsilon_r) = \lambda/\epsilon_r$

6-8 (6.8) PARALLEL-PLATE CAPACITOR



a) We can treat this problem as if we had two capacitors in series :

$C_1 = \epsilon_o A/(s-t)$, $C_2 = \epsilon_r \epsilon_o A/t$

$$C = C_1 C_2/(C_1+C_2)$$

$$= \left(\frac{\epsilon_o A}{s-t}\,\frac{\epsilon_r\epsilon_o A}{t}\right)\Bigg/\left[\epsilon_o A\left(\frac{1}{s-t}+\frac{\epsilon_r}{t}\right)\right]$$

$$= \frac{\epsilon_r\epsilon_o A}{t(s-t)\left(\frac{1}{s-t}+\frac{\epsilon_r}{t}\right)} = \frac{\epsilon_r\epsilon_o A}{t+(s-t)\epsilon_r} = \frac{\epsilon_o A}{s-t+t/\epsilon_r}$$

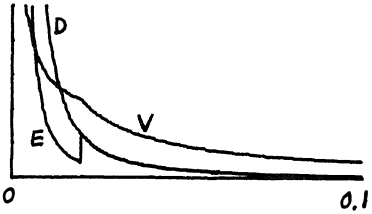
b) $C/C_0 = \dfrac{1}{1-(t/s)(2/3)}$

6-9 (6.7)

a) Since the only free charge is Q, Eq. 6-17 gives us that

$D = Q/4\pi r^2 = 10^{-9}/4\pi r^2 = 7.96 \times 10^{-11}/r^2$ C/m^2

both inside and outside the dielectric.



Inside the dielectric,

$E_i = D/\epsilon_r\epsilon_o = Q/4\pi\epsilon_r\epsilon_o r^2$

$= 7.95 \times 10^{-11}/3r^2 = 2.65 \times 10^{-11}/r^2$

Outside the dielectric,

$E_o = D/\epsilon_o = Q/4\pi\epsilon_o r^2 = 8.94/r^2$

To find V, we set V = 0 at infinity.

Outside the sphere, $V_o = Q/4\pi\epsilon_o r = 9.00/r$ V

At the surface of the sphere, V = 450 V.

Inside the sphere, $V_i = 450 + \displaystyle\int_r^R (Q/4\pi\epsilon_r\epsilon_o r^2)dr = 300 + 3.00/r$

b) $\sigma_b = P = D - \epsilon_o E = (Q/4\pi\epsilon_o R^2)(1 - 1/\epsilon_r) = 1.33 \times 10^{-7}$C/m^2

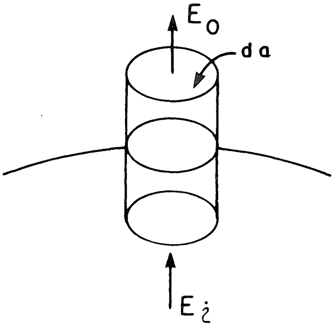
c) Let us apply Gauss's law to a small element of volume at the surface. The bound charge on the element of area da is

$\sigma_b da = \epsilon_o(E_o da - E_i da)$,

$\sigma_b = \epsilon_o(E_o - E_i) = (Q/4\pi R^2)(1-1/\epsilon_r)$,

as previously.

The discontinuity in E is due to the bound surface charge.



6-10 (6.7) CHARGED DIELECTRIC SPHERE

Outside the sphere, $E = (4/3)\pi R^3\rho_f/4\pi\epsilon_o r^2 = R^3\rho_f/3\epsilon_o r^2$

At $r = R$, $V = (4/3)\pi R^3\rho_f/4\pi\epsilon_o R = R^3\rho_f/3\epsilon_o R = R^2\rho_f/3\epsilon_o$

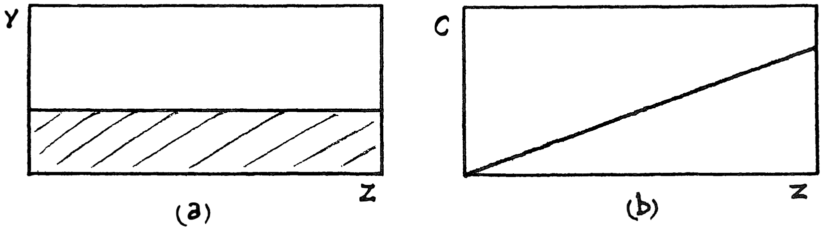
Inside the sphere, $D = (4/3)\pi r^3\rho_f/4\pi r^2 = r\rho_f/3$, $E = r\rho_f/3\epsilon_r\epsilon_o$

At the center, $$V = R^2\rho_f/3\epsilon_o + \int_0^R (r\rho_f/3\epsilon_o\epsilon_r)dr = (R^2\rho_f/3\epsilon_o)(1+1/2\epsilon_r)$$

6-11 (6.7) MEASURING SURFACE CHARGE DENSITIES ON DIELECTRICS

Reference: Journal of Physics E 2, 412 (1969).

6-12 (6.7) VARIABLE CAPACITOR UTILIZING A PRINTED CIRCUIT BOARD



In all cases, $dC = \epsilon_r\epsilon_o y dz/t$, $dC/dz = \epsilon_r\epsilon_o y/t$

$$y = \frac{10^{-3}}{3 \times 8.85 \times 10^{-12}}\frac{dC}{dz} = \frac{10^9}{26.55}\frac{dC}{dz}$$

a) $y = \dfrac{10^9}{26.55} 10^{-9} = 37.7$ mm

b) $y = \frac{10^9}{26.55} 10^{-8} \times 2z = 0.753z$ m

6-13 (6.8) EQUIPOTENTIAL SURFACES

a) No.

b) $\sigma_f = D$. The charge density is positive on the side where the vector $\vec{D}$ points away from the sheet.

6-14 (6.8) NON-HOMOGENEOUS DIELECTRICS

$\nabla \cdot \vec{D} = \nabla(\epsilon_r \epsilon_o \vec{E}) = \epsilon_o(\epsilon_r \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \epsilon_r) = 0$

Since $\nabla \epsilon_r \neq 0$, $\nabla \cdot \vec{E} \neq 0$ and $\rho_b \neq 0$

$\rho_b = \epsilon_o \nabla \cdot \vec{E} = -(1/\epsilon_r)\vec{E} \cdot \nabla \epsilon_r$

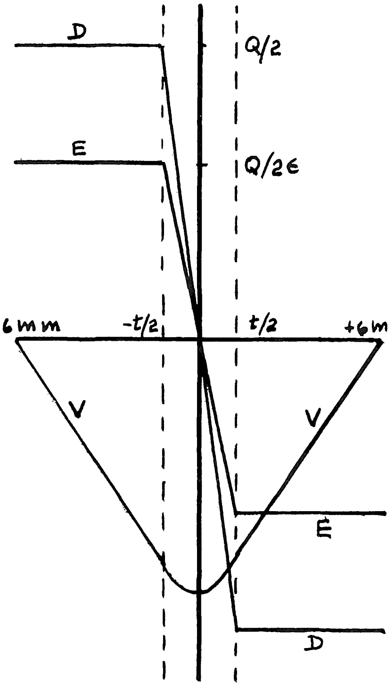
6-15 (6.9) FIELD OF A SHEET OF ELECTRONS TRAPPED IN LUCITE

a) The total free charge is -10^{-7}C,

$\rho_f = -10^{-7}/25 \times 10^{-4} \times 2 \times 10^{-3}$

$= -2.000 \times 10^{-2}$ C/m^3

$\rho_b = -(1-1/\epsilon_r)\rho_f = 1.375 \times 10^{-2}$ C/m^3



b) From Gauss's law, D on each side, in the neutral region, is one half the free charge per square meter:

$D_n = -(1/2)10^{-7}/25 \times 10^{-4}$

$= -2.00 \times 10^{-5}$ C/m^2

$P_n = (1-1/\epsilon_r)D_n = -1.375 \times 10^{-5}$ C/m^2

$E_n = (D_n - P_n)/\epsilon_o = D_n/\epsilon_r \epsilon_o$

$= -7.062 \times 10^5$ V/m

Since $E_n = -dV_n/dx$, and since

$V_n = 0$ at $x = 6 \times 10^{-3}$,

$V_n = 7.062 \times 10^5 x - 4{,}237$ V

c) At both surfaces, $\vec{P}$ points inward, like $\vec{E}$ and $\vec{D}$, and

$\sigma_b = P_n = -1.375 \times 10^{-5}$ C/m^2

d) Inside the charged region,

$\nabla\cdot\vec{D}_c = dD_c/dx = \rho_f = -2.000 \times 10^{-2}$ C/m^3, $D = -2.000 \times 10^{-2} x$ C/m^2

The constant of integration is zero because D changes sign at x = 0. So D = 0 at x = 0. Also,

$\nabla\cdot\vec{E}_c = dE_c/dx = (\rho_f + \rho_b)/\epsilon_o = -7.062 \times 10^8$ V/m^2, $E_c = -7.062 \times 10^8 x$ V/m

The constant of integration is again zero, for the same reason.

From Poisson's equation,

$\nabla^2 V_c = d^2 V_c/dx^2 = -(\rho_f + \rho_b)/\epsilon_o = 7.062 \times 10^8$ V/m^2

$dV_c/dx = 7.062 \times 10^8 x$ V/m^2

The constant of integration is zero, since $dV_c/dx = -E_c$. Thus

$V_c = 3.531 \times 10^8 x^2 - 3\ 884$ V

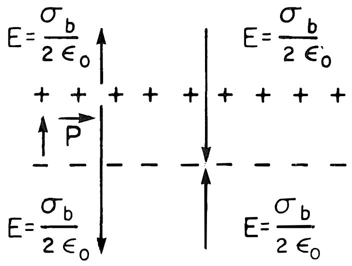
The constant of integration is now chosen to make $V_c = V_n$ at $x = 10^{-3}$.

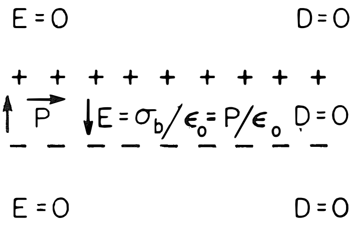
e) See curves.

f) The stored energy is $2 \times (1/2) \int_0^{10^{-3}} \rho_f V_c 25 \times 10^{-4} dx = 1.883 \times 10^{-4}$ J

g) No.

6-16 (6.10) SHEET ELECTRET





6-17 (6.7) RELATION BETWEEN R AND C FOR ANY PAIR OF ELECTRODES

Let the area of one plate be S and the spacing s. Then

$C = \epsilon_r \epsilon_o S/s$, $R = s/\sigma S$, $RC = \epsilon_r \epsilon_o/\sigma$

CHAPTER 7

7-1 (7.1) CONTINUITY CONDITIONS AT AN INTERFACE

$D = \lambda/2\pi r$, both inside and outside the dielectric,

$E_i = \lambda/2\pi\epsilon_r\epsilon_o r$ inside,

$E_o = \lambda/2\pi\epsilon_o r$ outside.

Thus, at the surface, $E_o - E_i = (\lambda/2\pi\epsilon_o R)(1-1/\epsilon_r)$.

V is continuous at the surface, but its slope dV/dr is smaller inside than outside.

7-2 (7.1) CONTINUITY CONDITIONS AT AN INTERFACE

$D = Q/4\pi r^2$, both inside and outside the dielectric,

$E_i = Q/4\pi\epsilon_r\epsilon_o r^2$ inside,

$E_o = Q/4\pi\epsilon_o r^2$ outside.

Thus, at the surface, $E_o - E_i = (Q/4\pi\epsilon_o R^2)(1-1/\epsilon_r)$.

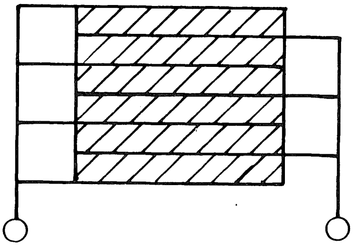
V is continuous at the surface, but its slope is smaller inside the dielectric.

7-3 (7.2) ENERGY STORAGE IN CAPACITORS

$W = QV/2 = CV^2/2 = 10^{-6} \times 10^6/2 = 0.5$ J

$mgh = 0.5$ J, $h = 0.5/1 \times 9.8 = 51$ mm

7-4 (7.2) ENERGY STORAGE IN CAPACITORS



For Mylar,

$W_1 = 3.2 \times 8.85 \times 10^{-12}(1.5 \times 10^8)^2/2$

$= 3.2 \times 10^5$ J/m^3

One would use the geometry shown in the figure. We need an absolute minimum of one kilowatt-hour. Then we need 3,600 x 1 000, or 3.6×10^6 J. Assuming 100 % efficiency, which is unrealistic (the actual overall efficiency might be, say 25 %), the capacitor would have a volume of 11 m^3. The density of Mylar being approximately equal to that of water, the capacitor would have a mass of 11 tons, which is absurd !

7-6 (7.1) BOUND SURFACE CHARGE DENSITY

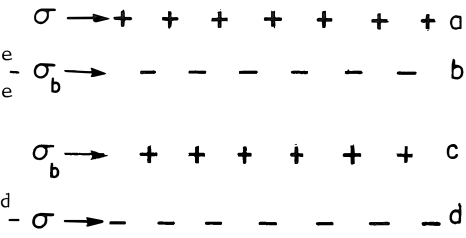
$D_B = \epsilon_o E_B + P_B$, $\sigma_B = P_B = D_B - \epsilon_o E_B = \epsilon_r \epsilon_o V/s - \epsilon_o V/s = (\epsilon_r - 1)\,\epsilon_o V/s$

7-7 (7.3) EXAMPLE OF A LARGE ELECTRIC FORCE

The force per square meter is $\epsilon_r \epsilon_o E^2/2 = 35 \times 8.85 \times 10^{-12} (4 \times 10^7)^2/2$ $= 2.5 \times 10^5$ Pa.

The force is 2.5 atmospheres.

7-8 (7.3) PERPETUAL-MOTION MACHINE



We have four sheets of charge as in the figure. Sheets a and b are coincident and are situated in the fields of c and d. Choosing the right-hand direction as positive, the field at the position of a and b is

$E_{a,b} = \sigma/2\epsilon_o - \sigma_b/2\epsilon_o = (1/2\epsilon_o)$

$[\sigma - (1 - 1/\epsilon_r)\sigma] = \sigma/2\epsilon_r \epsilon_o$

Then the forces per unit area on a and b are

$F_a = \sigma^2/2\epsilon_r \epsilon_o$, $F_b = -\sigma\sigma_b/2\epsilon_r \epsilon_o = -(1 - 1/\epsilon_r)\sigma^2/2\epsilon_r \epsilon_o$

Similarly,

$E_c = \sigma/2\epsilon_o - \sigma_b/2\epsilon_o + \sigma/2\epsilon_o = (1/2\epsilon_o)[2\sigma - (1 - 1/\epsilon_r)\sigma] = (1 + 1/\epsilon_r)\sigma/2\epsilon_o$

$F_c = (1 + 1/\epsilon_r)\sigma\sigma_b/2\epsilon_o = (1 + 1/\epsilon_r)(1 - 1/\epsilon_r)\sigma^2/2\epsilon_o = (1 - 1/\epsilon_r^2)\sigma^2/2\epsilon_o$

$E_d = \sigma/2\epsilon_o$, $F_d = -\sigma^2/2\epsilon_o$

Finally,

$F_a + F_b + F_c + F_d = 0$

7-9 (7.3) SELF-CLAMPING CAPACITOR

$F = (\epsilon_o E^2/2)S = (\sigma^2/2\epsilon_o)S = (S/2\epsilon_o)(VC/S)^2 = (S/2\epsilon_o)(V^2/S^2)(\epsilon_r \epsilon_o S/t)^2$

$= \epsilon_r^2 \epsilon_o S V^2/2t^2 = 3^2 \times 8.85 \times 10^{-12} \times 4.38 \times 10^{-2} \times 36 \times 10^8 /$

$2 \times (7.62 \times 10^{-4})^2 = 1.1 \times 10^4$ N

This is a very large force. It is approximately the weight of a mass of one ton.

7-10 (7.3) ELECTROSTATIC CLAMPS

a) $(1/2)\epsilon_r^2\epsilon_o(V/d)^2 = 2 \times 10^5$ Pa, $d = 15$ μm

b) The E in the Mylar is $3000/1.5 \times 10^5$ V/m. Then the E in the air is $3.2 \times 3000 \times 10^5/1.5 = 6.4 \times 10^8$ V/m

c) $(1/2)\epsilon_r\epsilon_o(V/d)^2 = 2 \times 10^5$ Pa, $d = 8.4$ μm

Reference: Static Electrification 1975, p. 215.

7-11 (7.3) CALCULATING AN ELECTRIC FORCE BY THE METHOD OF VIRTUAL WORK

Let the force be F. Assume a virtual displacement dx. Then the work done by the battery is equal to the mechanical work done, plus the increase in the stored energy, these two quantities being equal. Thus

$Fdx = d(QV/2) = d(V^2C/2) = V^2dC/2 = (V^2/2)\epsilon_r\epsilon_o \ell dx/s$

$F = \epsilon_r\epsilon_o \ell V^2/2s = 3 \times 8.85 \times 10^{-12} \times 0.1 \times 10^6/2 \times 10^{-3} = 1.33 \times 10^{-3}$N

7-12 (7.4) ELECTRIC FORCE

$\nabla E^2 = (\partial/\partial x)E^2\vec{i} + (\partial/\partial y)E^2\vec{j} + (\partial/\partial z)E^2\vec{k} = 2E\,\partial E/\partial x\,\vec{i} + \ldots = 2E\nabla E$

7-13 (7.4) ELECTRIC FORCE

See Prob. 7-10.

The mechanical work done is equal to the increase in electric energy. Both energies are supplied by the battery.

$Fdx = d(V^2C/2) = V^2dC/2 = (V^2/2)(\epsilon_r-1)\epsilon_o \ell dx/s$

$F = (V^2/2)(\epsilon_r-1)\epsilon_o \ell/s = (10^6/2)(3-1)8.85 \times 10^{-12} \times 0.1/10^{-3}$

$= 8.85 \times 10^{-4}$N

7-15 (7.4) ELECTRIC FORCE ON A DIELECTRIC

From Gauss's law, $E = (\lambda/2\pi\epsilon_r\epsilon_o)/r$

From Prob. 4-6, $V = (\lambda/2\pi\epsilon_r\epsilon_o)\ell n(R_2/R_1)$

Thus $E = V/r\ell n(R_2/R_1)$, $dE^2/dr = [V/\ell n(R_2/R_1)]^2(-2/r^3)$

The force is directed inwards. Disregarding the sign,

$F' = (\epsilon_r-1)\epsilon_o V^2/\ell n^2(R_2/R_1)r^3 = 8.85 \times 10^{-12} \times 1.5 \times 625 \times 10^6/\ell n^2 5 \times r^3$

$= 3.2 \times 10^{-3}/r^3$N/m^3

b) Near the inner conductor, $F' = 3.2 \times 10^{-3}/10^{-9} = 3.2 \times 10^{6}\,N/m^{3}$
The gravitational force per cubic meter is $9.8 \times 10^{3}\,N/m^{3}$. So
(Electric force)/(Gravitational force) $= 3.2 \times 10^{6}/9.8 \times 10^{3} = 330$

7-16 (7.6) DISPLACEMENT AND POLARIZATION CURRENTS

From Sec. 5.14 the voltage on the capacitor is

$V_C = V[1-\exp(-t/RC)]$, $E = V_C/s$

$D = \epsilon_r\epsilon_o E = \epsilon_r\epsilon_o(V/s)[1-\exp(-t/RC)]$

$dD/dt = \epsilon_r\epsilon_o(V/s)(1/RC)\exp(-t/RC)$

$P = (\epsilon_r-1)\epsilon_o E = (\epsilon_r-1)\epsilon_o(V/s)[1-\exp(-t/RC)]$

$dP/dt = [(\epsilon_r-1)\epsilon_o V/RCs]\exp(-t/RC)$

7-17 (7.7) DIRECT ENERGY CONVERSION

$C_1 = \epsilon_r\epsilon_o A/s = 8000 \times 8.5 \times 10^{-12}/2 \times 10^{-4} = 340\ \mu F$

$Q_1 = C_1 V_B = 340 \times 10^{-6} \times 700\ C$

$W_e = Q_1V_2/2 - Q_1V_1/2 = 0.248\ (3500-700)/2 = 347\ J$

$W_{th} = 2.9 \times 10^{6} \times 2 \times 10^{-4} \times 30 = 1.74 \times 10^{4}\ J$

$W_e/W_{th} = 347/1.74 \times 10^{4} = 0.02$

The efficiency is only 2 percent.

References: S.L. Soo, Direct Energy Conversion, p 184; Proc. IEEE, 51, 838 (1963).

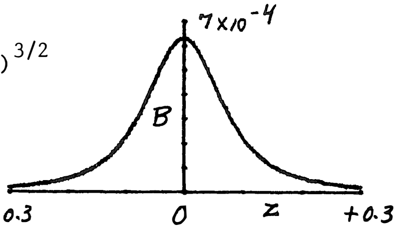
CHAPTER 8

8-1 (8.1) MAGNETIC INDUCTION ON THE AXIS OF A CIRCULAR LOOP

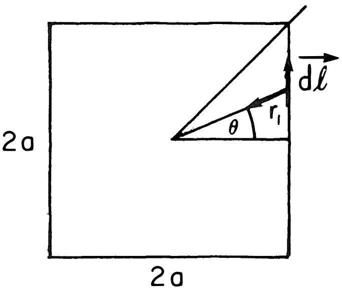
$B = \mu_o NIa^2/2(a^2 + z^2)^{3/2}$

$= 4\pi \times 10^{-7} \times 100 \times 1 \times 0.1^2/2(0.01xz^2)^{3/2}$

$= 2\pi \times 10^{-7}/(0.01+z^2)^{3/2}$



8-2 (8.1) SQUARE CURRENT LOOP



$$B = 8(\mu_o I/4\pi)\int_0^a d\ell\cos\theta/(a^2+\ell^2)$$

$$= 8(\mu_o I/4\pi)\int_0^a a\ d\ell/(a^2+\ell^2)^{3/2}$$

$$= 2^{\frac{1}{2}}\mu_o I/\pi a$$

8-3 (8.1) FIELD OF A CHARGED ROTATING DISK

a) $E = \sigma/\epsilon_o$, b) $\alpha = v\sigma = \omega r\sigma$

c) A ring of radius r and width dr acts as a current loop. So, from Sec. 8.1.2,

$$B = \int_0^R \mu_o(\omega r\sigma dr)/2r = \mu_o\omega R\sigma/2$$

d) $E = 10^{-6}/8.85\times10^{-12} = 1.13\times10^{5}\,V/m$

$B = 0.5\times4\pi\times10^{-7}\times10^{3}\times0.1\times10^{-6} = 6.28\times10^{-11}\ T$

8-4 (8.1) SUNSPOTS

a) The current loop between r and r + dr carries a current $2\pi r dr\sigma(\omega/2\pi) = \omega r\sigma dr$. At the center,

$$B = (\mu_o/2)\int_0^R \omega r\sigma dr/r = \mu_o\omega R\sigma/2$$

$\sigma = 2B/\mu_o\omega R = 2\times0.4/4\pi\times10^{-7}\times3\times10^{-2}\times10^{7} = 20/3\pi$

The electron density is $(20/3\pi)/1.6\times10^{-19} \approx 10^{19}\ m^{-2}$

b) The current is the total charge divided by the period:

$I = [\pi\times10^{14}\times(20/3\pi)]/(2\pi/\omega) \approx 3\times10^{12}A$

c) The negative charge of the electrons is neutralized by quasi-stationary positive ions.

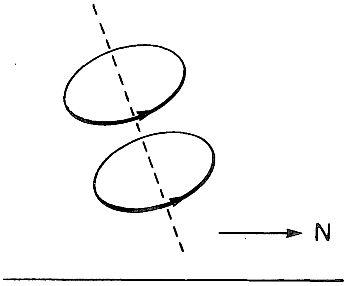
8-5 (8.1) HELMHOLTZ COILS

$B = 2\mu_o NIa^2/2(a^2+a^2/4)^{3/2} = (0.8)^{3/2}\mu_o NI/a = 8.992\times10^{-7}NI/a$

References: Durand, Magnétostatique, pp 44,270; O'Dell, The Electrodynamics of Magneto-Electric Phenomena, Appendix 4; Rubens, Rev. Sci

Instr. 16, 243 (1945)

8-6 (8.1) HELMHOLTZ COILS



a) In the northern hemisphere the magnetic field points downward. In a N-S plane, looking W, the coils are oriented as in the figure.

b) $a = 1$ m

c) $8.992 \times 10^{-7}\ NI/a = 5 \times 10^{-5}$,

$NI = 5 \times 10^{-5}/8.992 \times 10^{-7}$

$= 55.6$ At.

d) Try a current of 2 amperes so as to make the number of turns as small as possible. Then we need at least 28 turns in each coil. Then

$R = 28 \times 2\pi \times 1 \times 21.7 \times 10^{-3} = 3.82\Omega$

$V = 7.64V$, $P = 4 \times 3.82 = 15.3$ W for each coil.

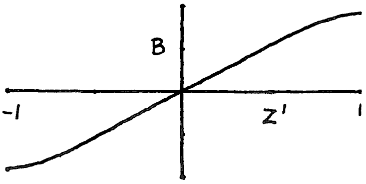
No cooling is required for this size of coil.

8-7 (8.1) LINEAR DISPLACEMENT TRANSDUCER

$$B = (\mu_o Ia^2/2)\left\{1/[a^2+(z-a)^2]^{3/2} - 1/[a^2+(z+a)^2]^{3/2}\right\}$$

$$= (\mu_o Ia^2/2)\ [1/(z^2-2az+2a^2)^{3/2} - 1/(z^2+2az+2a^2)^{3/2}]$$

$$= (\mu_o I/2a)[1/(z'^2-2z'+2)^{3/2} - 1/(z'^2+2z'+2)^{3/2}] \quad (z' = z/a)$$



8-8 (8.2) THE SPACE DERIVATIVES OF B IN A STATIC FIELD

$\nabla\cdot\vec{B} = \partial B_x/\partial x + \partial B_y/\partial y + \partial B_z/\partial z = 0$

$\partial B_y/\partial y$ is positive. By symmetry, $\partial B_x/\partial x$ is also positive. Then $\partial B_z/\partial z$ is necessarily negative.

8-9 (8.3) MAGNETIC MONOPOLES

$Q^*H\ell = 8.27 \times 10^{-15} \times (10/4\pi \times 10^{-7}) \times 0.16 = 1.05 \times 10^{-8}$ J

$= 1.05 \times 10^{-8}/1.6 \times 10^{-19} = 66$ Gev

8-10 (8.4) MAGNETIC FIELD OF A CHARGED ROTATING SPHERE

a) $\sigma = Q/4\pi r^2$, $V = Q/4\pi\epsilon_0 R$, $\sigma = \epsilon_0 V/R$

b) $\alpha = \sigma v = (\epsilon_0 V/R)\omega R \sin\theta = \epsilon_0 \omega V \sin\theta$

c) $B = \int_0^{\pi} \mu_0 (\epsilon_0 \omega V \sin\theta\, R d\theta)(R\sin\theta)^2/2R^3 = (2/3)\epsilon_0\mu_0\omega V$

d) $B = (2/3)8.85 \times 10^{-12} \times 4\pi \times 10^{-7} \times 2\pi \times (10^4/60) \times 10^4 = 7.75 \times 10^{-11}$ T

The field is parallel to the axis of rotation.

e) $m = (1/2) \int^{\pi} [(\epsilon_0 V/R)(2\pi R \sin\theta) R d\theta](\omega R \sin\theta) R \sin\theta$

$= \pi R^3 \epsilon_0 \omega V \int^{\pi} \sin^3\theta d\theta = (4/3)\pi R^3 \epsilon_0 \omega V$

f) $m = (4/3)\pi\ 10^{-3} \times 8.85 \times 10^{-12} \times 2\pi (10^4/60) 10^4 = 3.882 \times 10^{-7}$ Am^2

g) $(\pi/4)10^{-2} I = 3.882 \times 10^{-7}$, $I = 4.943 \times 10^{-5}$A

CHAPTER 9

9-1 (9.1) DEFINITION OF μ_0

$B = \mu_0 N'I = \mu_0$

9-2 (9.1) MAGNETIC FIELD OF A CURRENT-CARRYING TUBE

a) $B = \mu_0 I/2\pi r$

b) $\vec{A}$ is parallel to the tube and in the same direction as the current

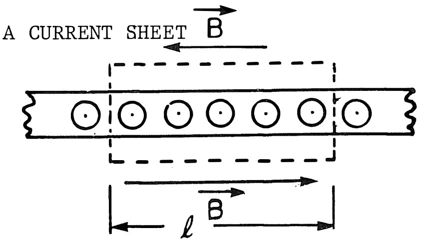
c) $\vec{B}$ inside is zero

d) $\vec{A}$ is as above. It is not zero. For any curve C, $\oint \vec{A}\cdot d\vec{\ell}$ gives the flux linkage. If the curve is entirely situated inside the tube where $\vec{B} = 0$, the integral is zero and $\vec{A}$ must be uniform. Its value is of no interest, since $\vec{B} = 0$.

9-3 (9.1) MAGNETIC FIELD CLOSE TO A CURRENT SHEET

Consider the dashed curve

$2B\ell = \mu_0 \alpha \ell$, $B = \mu_0 \alpha/2$



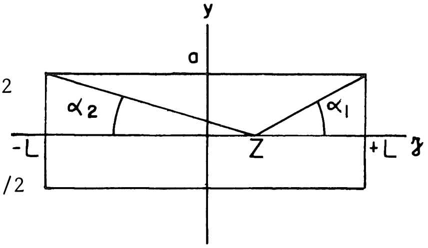
9-4 (9.1) VAN DE GRAAFF HIGH-VOLTAGE GENERATOR

a) $\sigma = 2\epsilon_o E = 2 \times 8.85 \times 10^{-12} \times 2 \times 10^{3}/10^{-3} = 3.54 \times 10^{-5}\ \text{C/m}^2$

$I = 3.54 \times 10^{-5} \times 0.5 \times \pi \times 0.1 \times 60 = 3.336 \times 10^{-4}\text{A}.$

b) $B = 4\pi \times 10^{-7} \times (3.336 \times 10^{-4}/0.5)/2 = 4\pi \times 3.336 \times 10^{-11} = 4.192 \times 10^{-10}\text{T}$

9-5 (9.1) SHORT SOLENOID



$$B = (\mu_o I a^2/2)\int_{-L}^{+L} (N/L)\,dz/[a^2+(z-Z)^2]^{3/2}$$

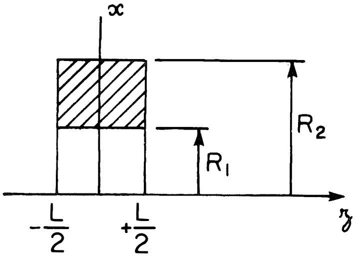
$$= (\mu_o N I a^2/2L)\int_{-L}^{+L} d(z-Z)/[a^2+(z-Z)^2]^{3/2}$$

$$= (\mu_o N I a^2/2L)\left[\frac{z-Z}{a^2\{a^2+(z-Z)^2\}^{3/2}}\right]_{-L}^{+L}$$

$$= (\mu_o N I/2L)\left\{\frac{L-Z}{[a^2+(L-Z)^2]^{3/2}} + \frac{L+Z}{[a^2+(L+Z)^2]^{3/2}}\right\}$$

$$= (\mu_o N I/2L)(\cos\alpha_1 + \cos\alpha_2)$$

9-6 (9.1) FIELD AT THE CENTER OF A COIL



a) $$B = (\mu_o n I/2)\int_{R_1}^{R_2}\int_{-L/2}^{+L/2} x^2\,dx\,dz/(x^2+z^2)^{3/2}$$

$$= (\mu_o n I/2)\int_{R_1}^{R_2}\left[\frac{x^2 z}{x^2(x^2+z^2)^{\frac{1}{2}}}\right]_{-L/2}^{+L/2} dx$$

$$= (\mu_o n I/2)\int_{R_1}^{R_2}\frac{L\,dx}{(L^2/4 + x^2)^{\frac{1}{2}}}$$

$$= (\mu_o n I L/2)\left[\ell n\{x+(L^2/4+x^2)^{\frac{1}{2}}\}\right]_{R_1}^{R_2} = (\mu_o n I L/2)\,\ell n\,\frac{R_2+(L^2/4+R_2^2)^{\frac{1}{2}}}{R_1+(L^2/4+R_1^2)^{\frac{1}{2}}}$$

$$= (\mu_o n I L/2)\,\ell n\frac{\alpha+(\alpha^2+\beta^2)^{\frac{1}{2}}}{1+(1+\beta^2)^{\frac{1}{2}}}$$

Note: Integrating first with respect to x would be much more difficult.

b) The number of turns is $L(R_2-R_1)n$, and the average length of one turn is $2\pi(R_1+R_2)/2$. Thus the length of the wire is

$$\ell = L(R_2-R_1)n\pi(R_1+R_2) = \pi\left(R_2^{\,2}-R_1^{\,2}\right)Ln = Vn,$$

where V is the volume of the winding. Also

$$\ell = 2\pi(\alpha^2-1)(L/2R_1)R_1^{\,3}n = 2\pi n(\alpha^2-1)\beta R_1^{\,3}$$

9-7 (9.1) CURRENT DISTRIBUTION GIVING A UNIFORM B

The field inside the hole is the same as if one had two superposed current distributions: a uniform current density throughout the cross-section, plus a current in the opposite direction in the hole. The current in the full cylinder is $I_a = IR^2/(R^2-a^2)$.

From Ampere's law, $B_{ax} = -\mu_o I_a y/2\pi R^2$, $B_{ay} = \mu_o I_a x/2\pi R^2$

The current in the small cylinder is $I_b = I_a(a^2/R^2)$

$$B_{bx} = \mu_o I_b y/2\pi a^2 = \mu_o I_a y/2\pi R^2, \quad B_{by} = -\mu_o I_a(x-b)/2\pi R^2$$

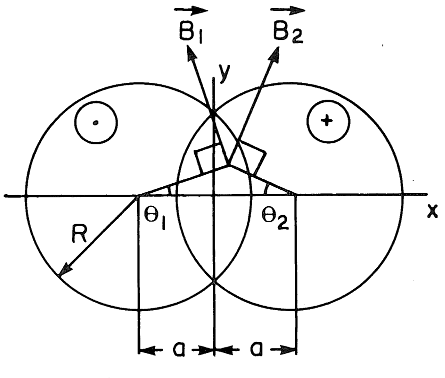
$$B_x = B_{ax} + B_{bx} = 0,$$

$$B_y = B_{ay} + B_{by} = \mu_o I_a b/2\pi R^2 = \mu_o Ib/2\pi(R^2-a^2)$$

The field is therefore uniform inside the hole.

Note that B is proportional to b. Thus B = 0 when b = 0, and B changes sign with b. Also, when $a \to R$, $b \to 0$ and B = 0.

9-8 (9.1) SADDLE COILS



This current distribution is obtained by superposing two full cylinders of current flowing in opposite directions.

Let I be the current flowing through the coil. Then the current I' that would flow through one complete circle is related to I as follows:

$$I = \left\{1 - (2/\pi)\left[\cos^{-1}(a/R) - (a/R)(1-a^2/R^2)^{\frac{1}{2}}\right]\right\}I'$$

Inside the left-hand circle, at the radius r, the B due to that side is

$B = \mu_o(I'/2\pi r)(r^2/R^2) = (\mu_o I'/2\pi)r/R^2$. So

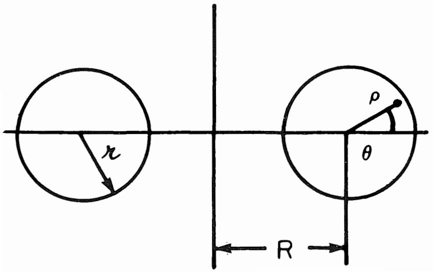
$$B_1 = (\mu_o I'/2\pi)[(a+x)^2 + y^2]^{\frac{1}{2}}/R^2, \quad B_2 = (\mu_o I'/2\pi)[(a-x)^2 + y^2]^{\frac{1}{2}}/R^2$$

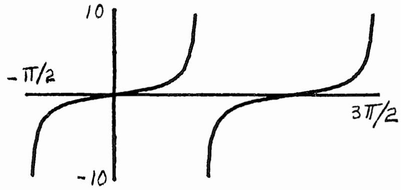
$$B_x = -B_1 \sin\theta_1 + B_2 \sin\theta_2 = -B_1 y/[(a+x)^2 + y^2]^{\frac{1}{2}} + B_2 y/[(a-x)^2 + y^2]^{\frac{1}{2}} = 0$$

$$B_y = B_1 \cos\theta_1 + B_2 \cos\theta_2 = B_1 (a+x)/[\,]^{\frac{1}{2}} + B_2 (a-x)/[\,]^{\frac{1}{2}} = \mu_o I' a/\pi R^2$$

So B is uniform and parallel to the y-axis.

9-9 (9.1) TOROIDAL COIL





$$\text{a)}\ \Phi = \iint B\cdot da = \int_0^r \int_0^{2\pi} \frac{\mu_o NI}{2\pi(R+\rho\cos\theta)}\, \rho\, d\rho\, d\theta$$

$$= (\mu_o NI/2\pi) \int_0^r \int_0^{2\pi} \frac{d\theta}{R+\rho\cos\theta}\, \rho\, d\rho$$

The integration with respect to θ must be done with care, taking into account the two branches of the curve. We integrate from $-\pi/2$ to $+\pi/2$, where $\cos\theta$ is positive, and then from $\pi/2$ to $3\pi/2$, where $\cos\theta$ is negative. Then

$$\int_0^{2\pi} \frac{d\theta}{R+\rho\cos\theta} = \int_{-\pi/2}^{\pi/2} \frac{d\theta}{R+\rho\cos\theta} + \int_{\pi/2}^{3\pi/2} \frac{d\theta}{R+\rho\cos\theta} = \int_{-\pi/2}^{\pi/2} \frac{d\theta}{R+\rho\cos\theta} + \int_{-\pi/2}^{\pi/2} \frac{d\theta}{R-\rho\cos\theta}$$

$$\int_{\pi/2}^{\pi/2} \frac{d\theta}{R+\rho\cos\theta} = \left[\frac{2}{(R^2-\rho^2)^{\frac{1}{2}}} \text{ arc tan}\left\{\frac{R-\rho}{(R^2-\rho^2)^{\frac{1}{2}}} \tan(\theta/2)\right\}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{4}{(R^2-\rho^2)^{\frac{1}{2}}} \text{ arc tan } \frac{R-\rho}{(R^2-\rho^2)^{\frac{1}{2}}}$$

$$\int_{-\pi/2}^{\pi/2} \frac{d\theta}{R-\rho\cos\theta} = \frac{4}{(R^2-\rho^2)^{\frac{1}{2}}} \text{ arc tan } \frac{R+\rho}{(R^2-\rho^2)^{\frac{1}{2}}}$$

$$\int_{0}^{2\pi} \frac{d\theta}{R+\rho\cos\theta} = \frac{4}{(R^2-\rho^2)^{\frac{1}{2}}} \left\{ \text{arc tan } \frac{R-\rho}{(R^2-\rho^2)^{\frac{1}{2}}} + \text{arc tan } \frac{R+\rho}{(R^2-\rho^2)^{\frac{1}{2}}} \right\}$$

Since arc tan a + arc tan b = arc tan$[(a+b)/(1-ab)]$,

$$\int_{0}^{2\pi} \frac{d\theta}{R+\rho\cos\theta} = \frac{4}{(R^2-\rho^2)^{\frac{1}{2}}} \left\{ \text{arc tan } \frac{2R/(R^2-\rho^2)^{\frac{1}{2}}}{1-(R^2-\rho^2)/(R^2-\rho^2)} \right\} = \frac{4(\pi/2)}{(R^2-\rho^2)^{\frac{1}{2}}} = \frac{2\pi}{(R^2-\rho^2)^{\frac{1}{2}}}$$

Thus $\Phi = (\mu_o NI/2\pi) \int_0^r \frac{2\pi}{(R^2-\rho^2)^{\frac{1}{2}}} \rho d\rho = \mu_o NI[R-(R^2-r^2)^{\frac{1}{2}}]$

The integration is more difficult with Cartesian coordinates.

b) $\overline{B} = \mu_o NI[R-(R^2-r^2)^{\frac{1}{2}}]/\pi r^2$ at the radius $\mathcal{R}$.

Set $\mathcal{R} = R + \rho \cos\theta$. Then $\mu_o NI/2\pi\mathcal{R} = \mu_o NI[R-(R^2-r^2)^{\frac{1}{2}}]/\pi r^2$

$\mathcal{R} = r^2/2[R-(R^2-r^2)^{\frac{1}{2}}] = r^2/2R[1-(1-r^2/R^2)^{\frac{1}{2}}]$

For $r^2 << R^2$, $\mathcal{R} \approx r^2/2R(r^2/2R^2) = R$

CHAPTER 10

There is an interesting article on the crossed-field mass spectrometer in The Journal of Physics E, Scientific Instruments, Volume 10 (1977) page 458.

10-1 (10.1) THE CYCLOTRON FREQUENCY

a) The centripetal force being BQv, $BQv = mv^2/R$.

b) Then $\omega = v/R = BQ/m$.

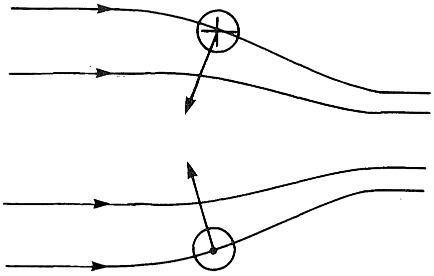
c) $BQ/2\pi m = 1 \times 1.6 \times 10^{-19}/2\pi \times 2 \times 1.7 \times 10^{-27} = 7.5$ megahertz.

10-2 (10.1) MOTION OF A CHARGED PARTICLE UN A UNIFORM B

The velocity component parallel to $\vec{B}$ is unaffected . The component normal to $\vec{B}$ gives a circular motion as in the preceding problem.

10-3 (10.1) MAGNETIC MIRRORS

The figure shows part of a helical orbit for a positive particle. The particle drifts to the right. The magnetic force points to the left. After a while, the drift velocity will also point to the left.



Reference. There is a good article on the magnetosphere in Contemporary Physics, 18, 165 (1977).

10-4 (10.1) HIGH ENERGY ELECTRONS IN THE CRAB NEBULA

a) $W = 2 \times 10^{14} \times 1.6 \times 10^{-19} = 3.2 \times 10^{-5}$ J.

b) $m = 3.2 \times 10^{-5}/9 \times 10^{16} = 3.6 \times 10^{-22}$ kg,

$m/m_o = mc^2/m_oc^2 = 3.6 \times 10^{-22}/9.1 \times 10^{-31} = 4 \times 10^8$

c) $R = mc/Be = 3.6 \times 10^{-22} \times 3 \times 10^8/2 \times 10^{-8} \times 1.6 \times 10^{-19} = 3.4 \times 10^{13}$m

d) $(2\pi \times 3.4 \times 10^{13}/3 \times 10^8)/(24 \times 3{,}600) = 8.2$ days

10-5 (10.1) MAGNETIC FOCUSING

a) An electron goes through one full circle in a time $T = 2\pi/\omega$. During that time it travels a distance $L = v_xT$. So

$L = (2eV/m)^{\frac{1}{2}}2\pi(m/Be) = 2^{3/2}\pi(mV/e)^{\frac{1}{2}}/B$, $B = 2^{3/2}\pi(mV/e)^{\frac{1}{2}}/L$

b) $B = 2^{3/2}\pi(9.1 \times 10^{-31} \times 10^4/1.6 \times 10^{-19})^{\frac{1}{2}}/0.5 = 4.24 \times 10^{-3}$ T

$IN' = B/\mu_o = 3373$ A

10-6 (10.1) DEMPSTER MASS SPECTROMETER

a) $mv^2/R = BQv$, $mv = BQR$, $(2m(1/2)mv^2)^{\frac{1}{2}} = BQR$

$(2mQV)^{\frac{1}{2}} = BQR$, $m = QR^2B^2/2V$

b) $B = (2mV/Q)^{\frac{1}{2}}/R$

For H_1^+, $B_1 = (2 \times 1.7 \times 10^{-27} \times 1000/1.6 \times 10^{-19})^{\frac{1}{2}}/0.06 = 7.7 \times 10^{-2}$T.

For H_2^+, $B_2 = 0.11$ T.

For H_3^+, $B_3 = 0.094$ T.

Note that $\Delta m/m = 2\Delta B/B$. Thus, for large m's, where $\Delta m/m$ becomes small from one isotope to the next, $\Delta B/B$ becomes even smaller.

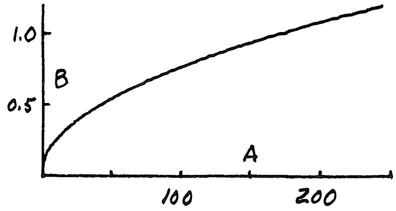
c)

$$B = (2mV/Q)^{\frac{1}{2}}/R$$

$$= (2x1.7x10^{-27}Ax1000/1.6x10^{-19})^{\frac{1}{2}}/0.06$$

$$= (3.4x10^{-5}/1.6)^{\frac{1}{2}}A^{\frac{1}{2}}/0.06 = 7.68x10^{-2}A^{\frac{1}{2}}$$

where A is the atomic weight. This value of m is approximate.



10-7 (10.1) MASS SPECTROMETER

$mv^2/R = BQv$, $R = mv/BQ$

$x = 2R = (2/B)(m/Q)v$

The time of flight from A to the target is $\pi R/v = \pi/B(m/Q)$. During that time the ion drifts through a distance

$y = (1/2)(QE/m)(\pi m/BQ)^2 = (\pi^2E/2B^2)(m/Q)$.

Reference: Rev. Sci. Instr. 45, 819 (1974).

10-8 (10.1) HIGH-TEMPERATURE PLASMAS

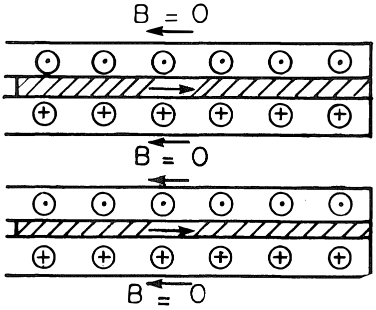
a) $mv^2/R = BQv$, $R = mv/BQ = (2mmv^2/2)^{\frac{1}{2}}/BQ \doteq 0.225$ m

b) A D^+ ion has the same velocity, but half the mass, so R = 113 mm.

Reference: Glasstone and Loveberg, Controlled Thermonuclear Reactions pages 156 and 395.

10-9 (10.1) HIGH TEMPERATURE PLASMAS

a) By symmetry, B can only be azimuthal. But the line integral of $\vec{B}\cdot d\vec{\ell}$ over a circle perpendicular to the paper and with its center on the axis of symmetry must be zero, since the net current is zero. Then B = 0. Similarly, B = 0 inside the inner cylinder.



b) See figure.

c) It bends downwards.

d) It bends upwards

e) They also return to the discharge.

Reference: Glasstone and Loveberg, Controlled Thermonuclear Reactions, p 278.

10-10 (10.1) ION BEAM DIVERGENCE

a) $I = v\lambda$, $\lambda = I/v$

b) $QE = Q(\lambda/2\pi\epsilon_o R) = QI/2\pi\epsilon_o Rv$

c) $QvB = Qv(\mu_o I/2\pi R) = QI\mu_o v/2\pi R$

d) $QE - QvB = (QI/2\pi R)(1/\epsilon_o v - \mu_o v) = (QI/2\pi\epsilon_o Rv)(1-\epsilon_o\mu_o v^2)$

10-11 (10.1) ION THRUSTER

See the solution to Prob. 2-14. Here, the force exerted on the ejected fuel, in the reference frame of the vehicle, is BIs, or m'v.

$P_D = I^2R = I^2(s/\sigma A)$, where A is the area of one of the electrodes, C or D.

$$\eta = 1/(1+P_D/P_G) = 1\bigg/\left(1 + \frac{I^2 s}{\sigma A}\,\frac{2}{BIsv}\right) = 1/(1 + 2BIs/\sigma B^2\tau v)$$

$$= 1/(1 + 2m'/\sigma B^2\tau)$$

Also, $2I/\sigma ABv = 2J/\sigma Bv = 2\sigma E/\sigma Bv = 2E/Bv$,

$\eta = 1/(1 + 2E/Bv)$.

As v increases, $\eta \to 1$, and $\eta \approx 1$ for $v >> 2E/B$.

10-12 (10.3) GAMMA

$(1-\beta^2)^{\frac{1}{2}} = 1/101$, $\beta^2 = 1 - 1/101$, $\beta = 0.99504$.

10-13 (10.5) REFERENCE FRAMES

$\gamma = 1/(1-1/4)^{\frac{1}{2}} = 1.155$

$x_2 = 1.155(1-1.5\text{x}10^8\text{x}1) = -1.732 \times 10^8$ m

$y_2 = y_1 = 1$ m, $z_2 = z_1 = 1$ m

$t_2 = 1.155(1-1/2\text{x}3\text{x}10^8) = 1.155$ s.

10-14 (10.5) REFERENCE FRAMES

$\gamma = 1.155$ as above

$x_1 = 1.155\ (1+1.5\text{x}10^8\text{x}1) = 1.732 \text{ x } 10^8\text{m},\ y_1 = z_1 = 1\text{m},\ t_1 = 1.155\text{ s}$

10-15 (10.8) HALL EFFECT

a) $\vec{v} = -M(\vec{E}+\vec{v}\text{x}\vec{B})$, where

$$\vec{v}\text{ x }\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix},$$

$$v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = -M\left(E_x\vec{i}+E_y\vec{j}+v_yB\vec{i}-v_xB\vec{j}\right),$$

$$v_x = -M(E_x+v_yB),\ v_y = -M(E_y-v_xB),\ v_z = 0.$$

b) $$v_x = -M[E_x-M(E_y-v_xB)B] = \frac{-ME_x+M^2BE_y}{1+M^2B^2} = \frac{M(-E_x+MBE_y)}{1+M^2B^2}$$

$$v_y = -M\left[E_y - \frac{MB(-E_x\ MBE_y)}{1+M^2B^2}\right]$$

$$= -M\left[\left(1 - \frac{M^2B^2}{1+M^2B^2}\right)E_y + \frac{MBE_x}{1+M^2B^2}\right] = -\frac{M}{1+M^2B^2}(E_y+MBE_x)$$

$$J_x = neM(E_x-ME_yB)/(1+M^2B^2)$$

$$J_y = neM(E_y+ME_xB)/(1+M^2B^2)$$

c) $V_y = (b/a)MV_xB = (10^{-3}/5\text{x}10^{-3})7\text{ x }1\text{ x }10^{-4} = 1.4\text{ x }10^{-4}\text{V}.$

d) When $E_y = 0$,

$$J_x = neME_x/(1+M^2B^2),\ I_x = bcneM(V_x/a)/(1+M^2B^2),$$

$$R = V_x/I_x = a(1+M^2B^2)/bcneM,$$

$$\Delta R/R_o = M^2B^2.$$

Let us calculate the mobility in copper. $J = \rho v = \sigma E$, $v = (\sigma/\rho)E$. Thus the mobility is $\sigma/\rho = \sigma/ne = 5.8\text{ x }10^7/8.5\text{ x }10^{28}\text{ x }1.6\text{ x }10^{-19} = 4.3\text{ x }10^{-3}$.

References: H.H. Wieder, Hall Generators and Magnetoresistors;

H. Weiss, Structure and Applications of Galvanomagnetic Devices.

10-17 (10.8) ELECTROMAGNETIC FLOWMETERS

$V = vBa$

CHAPTER 11

11-1 (11.1) BOAT TESTING TANK

a) $Bv\ell = 2 \times 10^{-5} \times 20 \times 3 = 1.2$ mV

b) Zero.

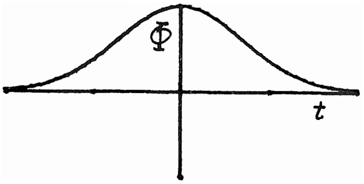
11-2 (11.1) EXPANDING LOOP

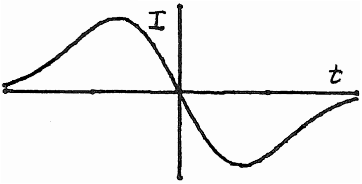
a) $I = Bvs/R$

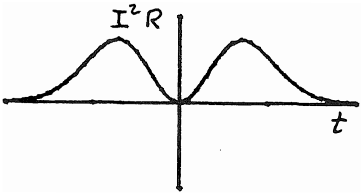
b) $(BIs)v = (Bvs)^2/R$

c) $I^2R = (Bvs)^2/R$. The power expended to move the bar appears as heat in the resistance R.

11-3 (11.2) INDUCED CURRENTS

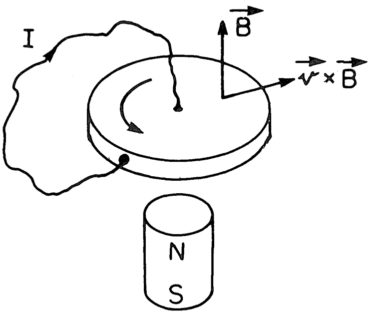






Reference: Rev. Sci. Instrum., 48, 1581 (1977).

11-4 (11.1) INDUCED CURRENTS



a) Counterclockwise

b) Counterclockwise

c) Since the flux linkage is constant, and since $\vec{v}\times\vec{B} = 0$, the induced electromotance is zero.

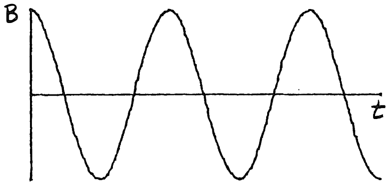
11-5 (11.2) INDUCED ELECTROMOTANCE

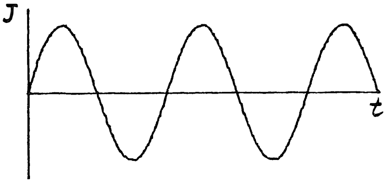
$$\mathcal{V} = d\Lambda/dt = NA(dB/dt) = 100 \times 10^{-2} \times 10^{-2} \times 2\pi \times 60 \sin(2\pi \times 60\, t)$$

$$= 3.77 \sin(2\pi \times 60\, t) V$$

We have disregarded the sign.

11-6 (11.2) ELECTROMAGNETIC PROSPECTION





a) The induced electromotance is azimuthal. Over a circle of radius r,

$2\pi r E = \pi r^2 B_o \omega \sin\omega t$, $E = (r B_o \omega/2)\sin\omega t$.

The induced current density is σE and is also azimuthal

$J = (\sigma r B_o \omega/2)\sin\omega t$.

b) With our sign convention, a positive J gives a positive B. At $t = 0$, $dB/dt = 0$ and $J = 0$. Then, as B decreases, J increases as per Lenz's law, etc.

11-7 (11.2) INDUCTION HEATING

a) $-d\Phi/dt = -\pi r^2 (d/dt)(\mu_o N' I_o \cos\omega t) = \mu_o \pi r^2 \omega N' I_o \sin\omega t$

b) The length of the conductor is $2\pi r$ and its cross-section is $L\,dr$.

Hence $R = 2\pi r/\sigma L dr$

c) $$dP_{av} = \frac{(\mu_o \pi r^2 \omega N' I_o)^2}{2x2\pi r/\sigma L dr} = \left[\mu_o^2 \pi \sigma \omega^2 N'^2 I_o^2/4\right] L r^3 dr,$$

The average value of $\sin^2 \omega t$ being equal to 0.5.

d) $$P_{av} = (\ )L \int_0^R r^3 dr = (\ )LR^4/4 = (4\pi \times 10^{-7} \times 2\pi \times 60 \times 5000 \times 2)^2$$

$$(\pi \times 10^5/16)(6 \times 10^{-2})^4 \times 1 = 5.71 \text{ W}$$

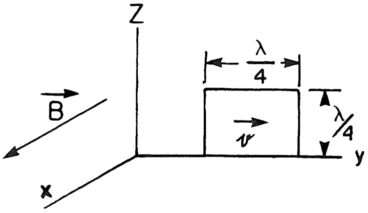
Note The power dissipated in the winding is I^2R, where R is its resistance. The conductivity of copper being 5.8×10^7 siemens per meter, if there are n layers, the cross-section of the wire is $\pi(n/5000)^2/4$ and $R = 2\pi \times 6 \times 10^{-2} \times 5000/[5.8 \times 10^7 \pi(n/5000)^2/4]$,

$I^2R \approx 10^3 I^2/n^2$.

If $n = 10$, $I^2R \approx 40$ W.

Reference: Standard Handbook for Electrical Engineers, p 22-28 and following.

11-8 (11.2) INDUCED ELECTROMOTANCE



$y = vt = (\omega\lambda/2\pi)t$, $\omega t = 2\pi y/\lambda$

$$\Phi = \int_y^{y+\lambda/4} B_o \sin(2\pi y/\lambda)\sin\omega t(\lambda/4)dy$$

$$= B_o \sin\omega t(\lambda/4)(\lambda/2\pi)\Big[-\cos(2\pi y/\lambda)\Big]_y^{y+\lambda/4}$$

$$= (\lambda^2/8\pi)B_o \sin\omega t[-\cos(2\pi y/\lambda + \pi/2) + \cos(2\pi y/\lambda)]$$

$$= (\lambda^2/8\pi)B_o(\sin^2\omega t + \sin\omega t\cos\omega t)$$

$$\mathcal{V} = |d\Phi/dt| = (\lambda^2/8\pi)B_o\omega(2\sin\omega t\cos\omega t - \sin^2\omega t + \cos^2\omega t)$$

$$= (B_o v\lambda/4)(\sin 2\omega t + \cos 2\omega t)$$

11-10 (11.4) THE TOLMAN AND BARNETT EFFECTS

In the reference frame of the conductor, the force on a particle of charge -e and mass m is

$\vec{F} = -e\vec{E} - m\vec{a} = -e\vec{E}'$, $\nabla \times \vec{E}' = \nabla \times \vec{E} + (m/e)\nabla \times \vec{a} = -\partial\vec{B}/\partial t + (m/e)\nabla \times \vec{a}$.

Reference: Landau and Lifshitz, Electrodynamics of Continuous Media, p 210.

11-11 (11.5) ELECTRIC CONDUITS

From the definition of $\vec{A}$ (Eq. 8-18), $\vec{A}$ is parallel to the wire. Then $\partial\vec{A}/\partial t$ is also parallel to the wire and, if there is a single wire, there is a longitudinal induced electromotance in the wire.

Reference: Standard Handbook for Electrical Engineers, Sec. 17-11.

11-12 (11.5) THE POTENTIALS V and A

Since $\nabla\times(\vec{A}-\nabla G) \equiv \nabla\times\vec{A}$, $\vec{B}$ is not affected. Also $-\nabla(V+\partial G/\partial t)-(\partial/\partial t)(\vec{A}-\nabla G) \equiv -\nabla V-\partial\vec{A}/\partial t$ and $\vec{E}$ is not affected either.

CHAPTER 12

12-1 (12.1)

$[H] = [\mu_o][L]$, $[\mu_o] = [H]/[L]$

12-2 (12.1) MUTUAL INDUCTANCE

Assume a current I in the wire. The flux linkage through the toroidal coil is

$$N\int_a^{a+b} \frac{\mu_o I}{2\pi r}\,b\,dr = (\mu_o IbN/2\pi)\ln(1+b/a),\quad M = (\mu_o bN/2\pi)\ln(1+b/a).$$

12-3 (12.1) MUTUAL INDUCTANCE

a) From Sec. 8.1.2, coil a produces at b a magnetic induction

$$B = \mu_o N_a I_a\, a^2/2(a^2+z^2)^{3/2}.$$

$$\text{So } M = \Phi_{ab}/I_a = \left\{\mu_o N_a a^2/2(a^2+z^2)^{3/2}\right\} N_b \pi b^2 = \pi\mu_o N_a N_b a^2 b^2/2(a^2+z^2)^{3/2}$$

b) M varies as the cosine of the angular displacement.

c) No.

12-4 (12.2) A OUTSIDE A SOLENOID

The magnetic flux inside the solenoid is $\pi R^2\mu_o N'I$. Then the electromotance induced in a loop of radius $r > R$ coaxial with the solenoid is $\pi R^2\mu_o N'dI/dt = 2\pi r\,dA/dt$, $A = (\mu_o/2r)N'R^2 I$.

12-5 (12.2) A INSIDE A SOLENOID

The magnetic flux inside a loop of radius $r < R$, coaxial with the solenoid, is

$$(d/dt)(\pi r^2 \mu_o N'I) = 2\pi r dA/dt, \quad A = (\mu_o/2)N'Ir.$$

12-6 (12.3) MAGNETIC MONOPOLES

The flux due to the current must exactly cancel that due to the monopoles. Then

$$LI = nNe^*, \quad I = nNe^*/L = 100 \times 1{,}200 \times 8.26 \times 10^{-15}/75 \times 10^{-3}$$

$$= 1.322 \times 10^{-8} A$$

12-7 (12.3) ROGOWSKI COIL

Reference: Glasstone and Loveberg, Controlled Thermonuclear Reactions, p 164; Rev. Sci. Instr. 42, 667 (1971).

12-8 (12.3) INDUCED CURRENTS

a) In the azimuthal direction, $R = 2\pi a/\sigma b\ell, B = \mu_o I/\ell$, $\Phi = \pi a^2 \mu_o I/\ell$,
$L = \Phi/I = \mu_o \pi a^2/\ell$.

b) $L = 4\pi \times 10^{-7} \times \pi \times 25 \times 10^{-6}/1 = 9.87 \times 10^{-11}$ H

$R = 2\pi \times 5 \times 10^{-3}/5.8 \times 10^7 \times 10^{-3} \times 1 = 5.42 \times 10^{-7} \Omega$

$L/R = 1.82 \times 10^{-4}$ s.

12-9 (12.3) COAXIAL LINES

In the annular region, B is due only to the current in the inner conductor, from Ampere's circuital law. Thus

$$B = \mu_o I/2\pi r, \quad \Phi' = (\mu_o I/2\pi)\int_{R_1}^{R_2} dr/r = (\mu_o I/2\pi)\ell n(R_2/R_1),$$

$$L' = (\mu_o/2\pi)\ell n(R_2/R_1).$$

12-10 (12.3) COAXIAL LINES

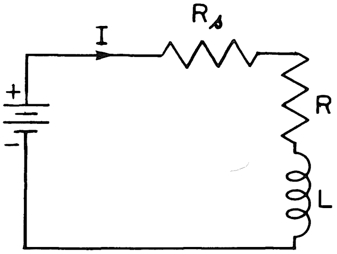
From Ampere's circuital law, the B in the annular region between the conductors is the same at all frequencies. Inside the conductors there is more field at lower frequencies. Hence W_m is larger at low frequencies and $L_{\ell f} > L_{hf}$.

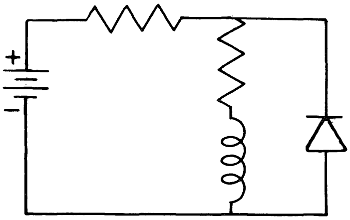
12-11 (12.5) LONG SOLENOID WITH CENTER TAP

$L_{AC} = (\mu_o N^2/\ell)\pi R^2$, $L_{AB} = L_{BC} = (\mu_o N^2/2\ell)\pi R^2$, $M = 0$.

Our formula for a long solenoid is based on the assumption that B is $\mu_o(N/\ell)I$ inside, and zero outside. With this assumption, the coupling coefficient k is zero, and M is zero.

12-13 (12.7) VOLTAGE SURGES ON INDUCTORS





a) $V = (R + R_s)I + LdI/dt \approx R_sI + LdI/dt$, $100 \approx R_sI - 10^4$

The voltages across R_s and L are both about 10^4 V.

b) Connect the diode as in figure b. Upon opening the switch, the current is $I = (V/R)e^{-Rt/L} = 10e^{-t}$.

12-14 (12.7) TRANSIENT IN RLC CIRCUIT

In circuit a, $V/V_s = 1-\exp(-t/RC)$

In circuit b, $L = 10^4 \times 10^{-6}/4 = 2.5 \times 10^{-3}$ H,

$Ld^2Q/dt^2 + RdQ/dt + Q/C = V_s$.

As in Prob. 12-15, the particular solution is $Q = V_sC = 10^{-4}C$,

$Ln^2 + Rn + 1/C = 0$, $n = -(R/2L) \pm (R^2/4L^2 - 1/LC)^{\frac{1}{2}}$,

$R/2L = 100/5 \times 10^{-3} = 2 \times 10^{4} = 1/LC$.

Thus the two values of n are equal and

$Q = (A + Bt)\exp(-Rt/2L) + V_sC$.

At $t = 0$, $Q = 0$ and $A = -V_sC$. Also,

$I = dQ/dt = \exp(-Rt/2L)[B - (R/2L)(A + Bt)]$.

At $t = 0$, $I = 0$ and $B = -(R/2L)V_sC$.

Finally, $Q = -V_sC(1+Rt/2L)\exp(-Rt/2L) + V_sC$

Since $R/2L = 2/RC$, $Q = V_sC[1-(1+2t/RC)\exp(-2t/RC)]$,

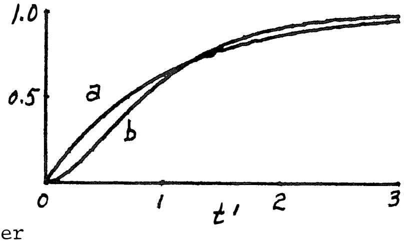
$V = Q/C$, $V/V_s = 1-(1+2t/RC)\exp(-2t/RC)$.

Summarizing,
$V/V_s = 1-\exp(-t/RC)$, for circuit a.

$V/V_s = 1-(1+2t/RC)\exp(-2t/RC)$, for circuit b.

Setting $t/RC = t'$,
$V/V_s = 1-\exp(-t')$, for a,
$V/V_s = 1-(1+2t')\exp(-2t')$, for b.

The figure shows Q/V_sC as a function of t for the two circuits. The charges are the same at t = 1.26 RC. The circuit with the inductor is slower at first, and then faster. The inductor is not useful.



<u>12-15</u> (12.7) TRANSIENT IN RLC CIRCUIT

a) $LdI/dt + RI = V$, $I = A\exp(-Rt/L) + V/R$

Since $I = 0$ at $t = 0$, $I = (V/R)[1-\exp(-Rt/L)] = 10[(1-\exp(-10t)]$.

b) $LdI/dt + RI + Q/C = V$, $Ld^2Q/dt^2 + RdQ/dt + Q/C = V$

The particular solution is $Q = VC = 10^{-4}$.

The complementary function is

$Q = Ae^{nt}$, $n = [-R\pm(R^2-4L/C)^{\frac{1}{2}}]/2L = -5 \pm 10^3 j$,

$Q = \exp(-5t)(B\cos 10^3t + D\sin 10^3t) + 10^{-4}$.

Since $Q = 0$ at $t = 0$, $B = -10^{-4}$,

$Q = \exp(-5t)(-10^{-4}\cos 10^3t + D\sin 10^3t) + 10^{-4}$,

$I = dQ/dt = \exp(-5t)(5 \times 10^{-4}\cos 10^3t + 10^{-1}\sin 10^3t - 5D\sin 10^3t + 10^3D\cos 10^3t)$

Also, $I = 0$ at $t = 0$. Then $D = -5 \times 10^{-7}$

$Q = \exp(-5t)(-10^{-4}\cos 10^3t - 5 \times 10^{-7}\sin 10^3t) + 10^{-4} \approx -10^{-4}\exp(-5t)\cos 10^3t$
$+ 10^{-4}$.

$I = -10^{-4}\exp(-5t)(-10^3\sin 10^3t) + 5 \times 10^{-4}\exp(-5t)\cos 10^3t$
$= 0.1\exp(-5t)\sin 10^3t + 5 \times 10^{-4}\exp(-5t)\cos 10^3t \approx 0.1\exp(-5t)\sin 10^3t$.

CHAPTER 13

13-1 (13.1) MAGNETIC FORCE

$BIL = 5 \times 10^{-2} \times 400 \times 5 \times 10^{-2} = 10^{-2}$ N

13-2 (13.1) MAGNETIC FORCE

a) Let the wire have a cross-section a, and let the current density be J. For a length of one meter,

$BJa = \rho ag$, $J = \rho g/B = 8.9 \times 10^{3} \times 9.8 \times 10^{-4} = 8.7 \times 10^{8}$ A/m^2

$I = 8.7 \times 10^{8} a$ A

b) $R' = 1/5.8 \times 10^{7} a$. Ω/m

$P' = (8.7 \times 10^{8} a)^2 / 5.8 \times 10^{7} a = 1.3 \times 10^{10} a$ W/m

If $a = 10^{-8}$, then $P' = 130$ W/m. The wire will become very hot. Convection will spoil the measurement.

c) In the Northern hemisphere there is a South magnetic pole. The lines of B point South. The current must point West.

d) At the poles the lines of B are vertical and the magnetic force on a horizontal wire is horizontal and perpendicular to the wire.

13-3 (13.1) MAGNETIC FORCE

$F = 50 \times 100 \times 0.5 \times 10^{-4} \times \sin 70^{\circ} = 0.235$ N

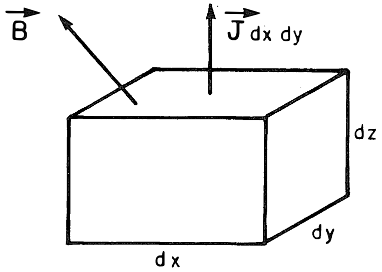
13-4 (13.1) MAGNETIC FORCE

$$\vec{F} = I_a \oint d\vec{\ell}_a \times \vec{B} = -I_a \vec{B} \times \oint d\vec{\ell}_a = 0.$$

13-5 (13.1) ELECTROMAGNETIC PUMPS

Consider an element of volume dxdydz, as in the figure, with the current flowing along the z-axis. The current is Jdxdy. Both $\vec{J}$ and $\vec{B}$ are uniform inside the infinitesimal element of volume. The force per unit volume is

$$(\vec{J}dxdy)\,dz \times \vec{B}/dxdydz = \sigma(\vec{E}+\vec{v}\times\vec{B})\times\vec{B} \ .$$



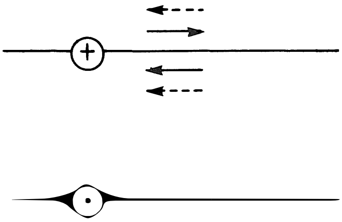
13-6 (13.1) HOMOPOLAR GENERATOR AND HOMOPOLAR MOTOR

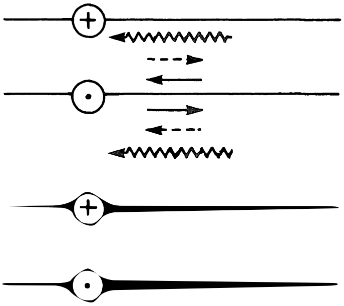
$$V = \int_0^R B\omega r dr = B\omega R^2/2 = 1 \times (3000 \times 2\pi/60) 0.25/2 = 39.27 \text{ V.}$$

13-7 (13.1) HOMOPOLAR MOTOR

The current has a radial component pointing towards the axis. The azimuthal component of the current gives a B pointing to the right. The wheel turns counterclockwise.

13-8 (13.2) MAGNETIC PRESSURE





a) We have precisely the situation described in Sec. 13.2.

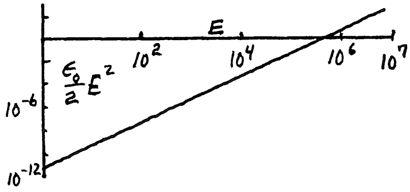
b) Inside the inner solenoid, there is zero magnetic field. Between the two solenoids the field is B. The magnetic pressure $B^2/2\mu_o$ pushes inward on the inner solenoid.

13-9 (13.2) MAGNETIC PRESSURE

a) $B^2/2\mu_o = B^2/2 \times 4\pi \times 10^{-7}$ Pa $\approx (B^2/2 \times 4\pi \times 10^{-7}) 10^{-5}$ atmospheres

$\approx 4\ B^2$ atmospheres.

b)



c) (i) The pressure is always equal to the energy density

(ii) The electric "pressure" we have considered is associated with the fact that lines of force are under tension. This "pressure" is always attractive. (We have not considered the repulsion betweeen electric lines of force, which gives a positive pressure of $\epsilon_o E^2/2$. For example, if we have two electric charges of the same sign, one can find the correct force of repulsion by integrating $\epsilon_o E^2/2$ over the plane half-way between the charges, where the lines of force clash).

(iii) The magnetic pressure we are concerned with here is associated with the lateral repulsion between lines of force. This pressure is repulsive. In Prob. 15-6 we are concerned with the tension in the lines of force, which gives an attractive "pressure" of $B^2/2\mu_o$.

(iv) In pratice, the electric "pressure" is nearly always negligible while magnetic pressure is often large. For example, a large E of 10^6V/m gives an electric "pressure" of 5P, while a large B of 1 T gives a magnetic pressure of 4×10^5P.

13-10 (13.2) MAGNETIC PRESSURE

a) $B = \mu_o I/2\pi R$ outside, $B = 0$ inside, from Ampere's circuital law. Thus $p_m = (1/2\mu_o)(\mu_o I/2\pi R)^2 = (\mu_o/8\pi^2)(I/R)^2$.

b) $p_m = (4\pi \times 10^{-7}/8\pi^2)(9 \times 10^8/25 \times 10^{-8}) = 5.73 \times 10^7 \text{Pa} \approx 5.73$ atmospheres.

Reference: J. Phys. D. Appld Phys. 6, 2187 (1973).

13-11 (13.2) MAGNETIC PRESSURE

a) $P = 1/8\pi \times 10^{-7} \approx 4 \times 10^5$ Pa ≈ 4 atmospheres.

b) The pressure would be unchanged, since B is uniform inside a long solenoid.

13-12 (13.3) ENERGY STORAGE

$\epsilon_o E^2/2 = 8.85 \times 10^{-12} \times 10^{12}/2 = 4.43 \text{ J/m}^3$

$B^2/2\mu_o = 1/8\pi \times 10^{-7} = 3.98 \times 10^5 \text{ J/m}^3$

13-13 (13.4) MAGNETIC PRESSURE

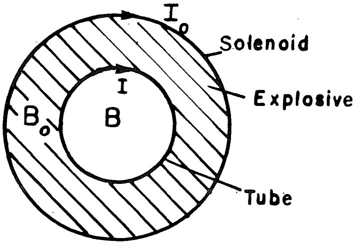
a) The magnetic force is $2\pi R \ell p_m$. It acts through a distance dR. Then the work done by the magnetic force is $2\pi R \ell p_m dR$.

b) The mechanical work done is

$$2\pi R\ell(B^2/2\mu_o)dR = 2\pi R\ell\left[(\mu_o NI/\ell)^2/2\mu_o\right]dR = \pi\mu_o I^2N^2RdR/\ell.$$

c) $I(Nd\Phi) = INd(\pi R^2\mu_o NI/\ell) = 2\pi\mu_o I^2N^2RdR/\ell$

13-14 (13.4) FLUX COMPRESSION



a) As the tube shrinks in diameter, an azimuthal current is induced that maintains the enclosed magnetic flux approximately constant. Hence $B \approx B_o(R_o/R)^2$.

b) $B = \mu_o\alpha,\ \alpha = 10^3/(4\pi\times10^{-7})$

$$\approx 10^9 \text{ A/m} \tag{1}$$

c) $$B \approx 10(10/1)^2 = 10^3 \text{ T}. \tag{2}$$

d) The solenoid maintains a constant B_o in its interior. The current in the tube increases the induction inside the tube to B. Thus the increase in magnetic energy is

$$\Delta W_m = \pi R^2L\left(B^2-B_o^2\right)\Big/2\mu_o = \pi R^2L\left(R_o^4\Big/R^4-1\right)B_o^2\Big/2\mu_o, \tag{3}$$

$$= \pi\times(10^{-4}/4)\times0.2(10^4-1)10^2/8\pi\times10^{-7} \approx 6\times10^6 \text{ J}. \tag{4}$$

The source supplies an extra energy

$$\Delta W_s = N\int_0^\infty I_o(d\Phi/dt)dt = NI_o\Delta\Phi = NI_o\pi R^2(B-B_o), \tag{5}$$

$$= NI_o\pi R^2\left(R_o^2\Big/R^2-1\right)B_o = \pi R^2(B_oL/\mu_o)\left(R_o^2\Big/R^2-1\right)B_o, \tag{6}$$

$$= (\pi/\mu_o)LB_o^2(R_o^2-R^2) = (\pi/4\pi\times10^{-7})0.2\times10^2(10^{-2}-10^{-4})/4$$

$$\approx 1.3\times10^5 \text{ J}. \tag{7}$$

The explosive supplies an energy

$$W_{expl} = -\int_{R_o}^{R} 2\pi rL\left[(B^2-B_o^2)/2\mu_o\right]dr = -(\pi L/\mu_o)\int_{R_o}^{R}\left(R_o^4\Big/r^4-1\right)B_o^2rdr, \tag{8}$$

$$= (\pi L/\mu_o)B_o^2\left(R_o^4/2R^2+R^2/2-R_o^2\right) \approx 6\times10^6 \text{ J}. \tag{9}$$

Note that $\Delta W_m = \Delta W_s + W_{expl}$.

Note also that, although the magnetic field just inside the solenoid is unaffected by the current I in the tube, the current I produces a $\partial A/\partial t$ in the solenoid that makes $W_s \neq 0$. The explosive supplies most of the energy. We have neglected the mechanical energy required to crush the tube, acoustic energy, etc.

13-15 (13.4) PULSED MAGNETIC FIELDS

a) $W_m = (B^2/2\mu_o)V = 4 \times 10^6$ J. Cost ≈ \$ 8×10^6

b) $W_m = 4 \times 10^6$ J $= 4 \times 10^6/3.6 \times 10^6 \approx 1$ kwh. Cost ≈ 2 to 10 cents, depending on prevailing rates.

c) $p = B^2/2\mu_o \approx 4 \times 10^9$ Pa $\approx 4 \times 10^4$ atmospheres.

13-16 (13.5) MAGNETIC ENERGY

a) $W_m = I_a\Lambda_a/2 + I_b\Lambda_b/2 = I_a(\Lambda_{aa} + \Lambda_{ba})/2 + I_b(\Lambda_{ab} + \Lambda_{bb})/2$

b) $W_m = L_aI_a^2/2 + MI_aI_b/2 + L_bI_b^2/2 + MI_aI_b/2 = L_aI_a^2/2 + L_bI_b^2/2 + MI_aI_b.$

13-17 (13.5) ENERGY STORAGE

a) $W_m = LI^2/2 = I(LI)/2 = I\Lambda/2$

b) $W_m = I(N\Phi)/2 = IN\pi R^2\mu_o NI/2\ell = \mu_o\pi R^2N^2I^2/2\ell$

13-18 (13.5) ENERGY STORAGE

a) i) $LdI/dt = V$, $I = (V/L)t$

ii) $(1/2)(Vt/L)Vt = (V^2/2L)t^2$

iii) $LI^2/2 = L(Vt/L)^2/2 = (V^2/2L)t^2$

b) i) $V = Q/C = (I/C)t$

ii) $(1/2)[(It/C)I]t = (I^2/2C)t^2$

iii) $CV^2/2 = C(It/C)^2/2 = (I^2/2C)t^2$

13-19 (13.4) ENERGY STORAGE

a) $\epsilon_r\epsilon_o E^2/2 = 3.2 \times 8.85 \times 10^{-12} \times 10^{16}/2 = 1.4 \times 10^5$ J/m^3

$B^2/2\mu_o = 64/2 \times 4\pi \times 10^{-7} = 2.5 \times 10^7$ J/m^3

b) $(\pi D^2/4)L \times 2.5 \times 10^7 = (\pi D^2/4)20\,D \times 2.5 \times 10^7 = 10^{10} \times 3{,}600 = 3.6 \times 10^{13}$,

$D = 45$ m, $L = 900$ m.

$2.5 \times 10^7 P = 250$ atmospheres

$A = \pi DL = \pi \times 45 \times 900 = 1.27 \times 10^5\ \mathrm{m}^2$

Reference: Foner and Schwartz, Superconducting Machines and Devices, p 41.

13-20 (13.6) SUPERCONDUCTING POWER TRANSMISSION LINE

a) $F = (\mu_o I/2\pi D)I = \mu_o I^2/2\pi D = 4\pi \times 10^{-7} \times (10^{11}/2 \times 10^5)^2/2\pi \times 5 \times 10^{-2}$

$= 10^6$ N/m

b) $B = 2\mu_o I/2\pi(D/2) = 2\mu_o I/\pi D = 8 \times 10^{-7} \times 5 \times 10^5/5 \times 10^{-2} = 8$ T.

$B^2/2\mu_o = 64/4\pi \times 10^{-7} = 5 \times 10^7\ \mathrm{J/m^3}$

c) $W = (1/2)LI^2 = (1/2)(6.6 \times 10^{-7} \times 10^6)(5 \times 10^5)^2 = 8.25 \times 10^{10}$ J

Cost $= 5 \times 10^{-3}(8.25 \times 10^{10}/3.6 \times 10^6) = \$\ 115.$

Reference: Proc. I.E.E.E., April 1967 page 57.

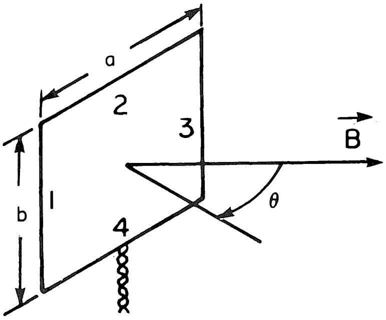
13-21 (13.8) ELECTRIC MOTORS AND MOVING-COIL METERS

a) $F_1 = NIBb$, perpendicular to both 1 and B

$F_2 = NIBa$, in the vertical direction

$F_3 = NIBb$, perpendicular to both 3 and B

$F_4 = NIBa$, in the vertical direction



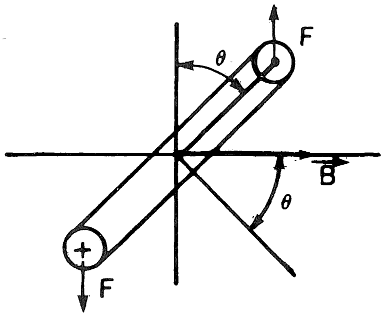
b) $T = 2NIBb(a \sin\theta/2) = NIBab \sin\theta$

13-22 (13.8) MAGNETIC TORQUE

a) $T = I\partial\Phi/\partial\theta = I\partial(BS \cos\theta)/\partial\theta$

$= ISB \sin\theta$

b) $\vec{T} = \vec{m} \times \vec{B}$



13-23 (13.8) ATTITUDE CONTROL FOR SATELLITES

a) $\Phi = NBA\cos\theta$, $T = I(\partial/\partial\theta)(NBA\cos\theta) = NIBA\sin\theta$

See figure for Prob. 13-22.

b) $IN = T/BA\sin\theta = 10^{-3}/4\times 10^{-5}\times(\pi/4)1.14^2\times 0.0873 = 280$ At

13-24 (13.9) MECHANICAL FORCES ON AN ISOLATED CIRCUIT

$$\Delta W_s = \int I(d\Phi/dt)\,dt = I\Delta\Phi$$

$$\Delta W_m = \Delta(LI^2/2) = (1/2)I^2\Delta L = (1/2)I^2\Delta(\Phi/I) = (1/2)I\Delta\Phi = (1/2)\Delta W_s$$

Thus the mechanical work, $\Delta W_s - \Delta W_m$, is equal to ΔW_m.

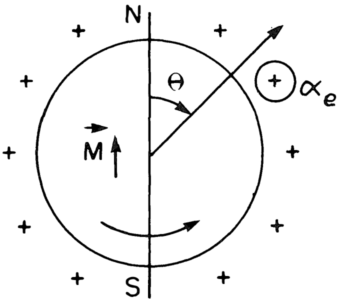
CHAPTER 14

14-1 (14.2) MAGNETIC FIELD OF THE EARTH

$\alpha_e = M\sin\theta$

If the sphere carried a surface charge density σ and rotated at an angular velocity ω, we would have

$\alpha = \sigma v = \sigma\omega R\sin\theta$, $\sigma\omega R = M$

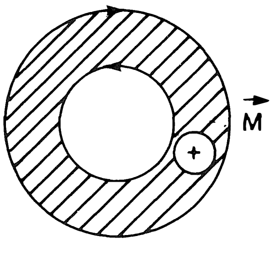


14-2 (14.3) EQUIVALENT CURRENTS

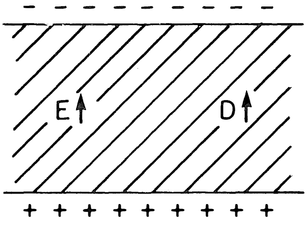
The surface current density is the same as if a toroidal coil were wound on the torus, and $\alpha_e = M$.

14-3 (14.4) EQUIVALENT CURRENTS

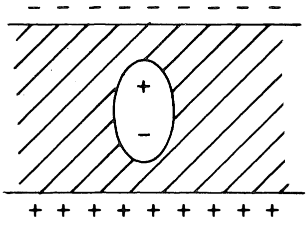
The equivalent currents are equal and in opposite directions. Thus $\vec{B} = 0$ inside the tube.



14-4 (14.4) DIELECTRICS AND MAGNETIC MATERIALS COMPARED



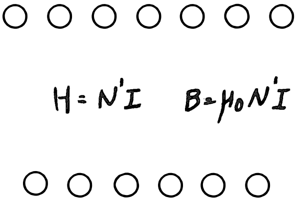
(a.)



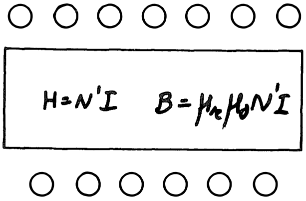
(b)

a) b) D is unaffected, E is reduced by ϵ_r. See Figs. a and b.

c) The energy is minimum.



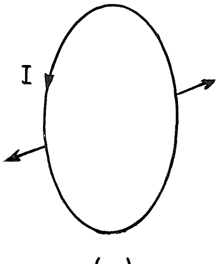
(c)



(d)

d) e) H is unaffected, B is increased by μ_r. See Figs. c,d

f) The energy is minimum. See Fig. e.
The loop is in stable equilibrium.



(e)

14-5 (14.4) MAGNETIC TORQUE

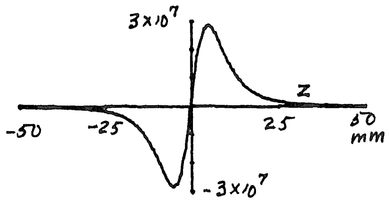
The magnet acts like a solenoid. See Probs 13-22 and 13-23.

14-6 (14.4) MEASUREMENT OF M

$B \propto \sin\theta\cos\theta \propto \sin 2\theta$

The field is largest at $\theta = 45$ degrees.

14-7 (14.4) MICROMETEORITE DETECTOR



a) $V = (d/dt)(MI_b)$

$$= (d/dt)\,\frac{\pi\mu_oN_a a^2 b^2}{2(a^2+z^2)^{3/2}}\,I_b$$

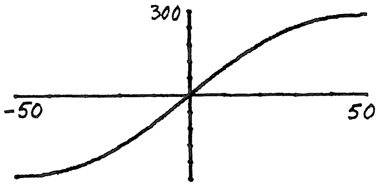
$$= (\mu_oN_a a^2/2)(\pi b^2 I_b)(d/dt)$$

$$[1/(a^2+z^2)^{3/2}] = (3/2)\mu_oN_a\;\frac{a^2}{(a^2+z^2)^{5/2}}\;zmv$$

b) See the figure

Reference: Rev. Sci. Instr. 42 663 (1971)

14-8 (14.4) MECHANICAL DISPLACEMENT TRANSDUCER



a) See curve.

b) $xz/(x^2+z^2)^{5/2} = 0.95\times10^4 z$,

$0.1/(0.01+z^2)^{5/2} = 9.5\times10^3$,

$z = 14.4$ mm

Reference: H. Wieder, Hall Generators and Magnetoresistors, p 95.

14-9 (14.5) MAGNETIZED DISK

Outside, we can use the field of a current loop:

$B_{ex} = \mu_oMta^2/2(a^2+z^2)^{3/2}$, $H_{ex} = Mta^2/2(a^2+z^2)^{3/2}$.

Inside, we have the same value of B, with $z^2 \ll a^2$:

$B_{in} = \mu_oMt/2a$, $H_{in} = B_{in}/\mu_o - M = M(t/2a-1) = -M(1-t/2a) \approx -M$

14-10 (14.6) TOROIDAL COIL WITH MAGNETIC CORE

Let N' be the number of turns per meter in both cases. From Eq. 14-15, H = N'I in both cases. With the air core, $B = \mu_oN'I$. With the magnetic core, B is larger by a factor μ_r.

The equivalent currents flow in the same direction as I.

14-11 (14.6) EQUIVALENT CURRENTS

a) $H = I/2\pi r$, $B = \mu_r\mu_oI/2\pi r$, $\Phi = \int_b^c (\mu_o\mu_rI/2\pi r)\ell dr = (\mu_r\mu_oI\ell/2\pi)\ell n(c/b)$

b) On the inner surface, $\alpha_e = M = B/\mu_o - H = (\mu_r - 1)H = \chi_m H = \chi_m I/2\pi b$,

in the same direction as the current

On the outer surface, $\alpha_e = \chi_m I/2\pi c$, in the opposite direction.

c) $B = \mu_o I/2\pi r$, as if the iron were absent.

14-12 (14.7) THE DIVERGENCE OF H

$\nabla\cdot\vec{B} = \nabla\cdot(\mu_r\mu_o\vec{H}) = \mu_o\nabla_{\mu_r}\cdot\vec{H} + \mu_r\mu_o\nabla\cdot\vec{H} = 0$

$\nabla\cdot\vec{H} \neq 0$ if $\nabla\mu_r \neq 0$ and if $\nabla\mu_r$ is not perpendicular to $\vec{H}$.

14-13 (14.8) THE MAGNETIZATION CURVE

Interpolating logarithmically between the points marked 2×10^3 and 2×10^4,

$\mu_r = 6.1 \times 2 \times 10^3 = 1.22 \times 10^4$.

14-14 (14.8) ROWLAND RING

a) $H = 500 \times 2.4/2\pi \times 0.2 \approx 1000$ A/m, $B \approx 0.5$ T

b) $V = Nd\Phi/dt = NSdB/dt = 10\pi \times 10^{-4} \times 0.5(10/2.4) = 6.6$ mV

14-15 (14.9) THE WEBER AMPERE-TURN

A weber is a unit of magnetic flux, and $d\Phi/dt$ is a voltage.

Thus a weber is a volt second. The number of turns is a pure number.

So [weber][ampere] = [volt second][ampere] = [watt][second] = [joule]

14-17 (14.9) TRANSFORMER HUM

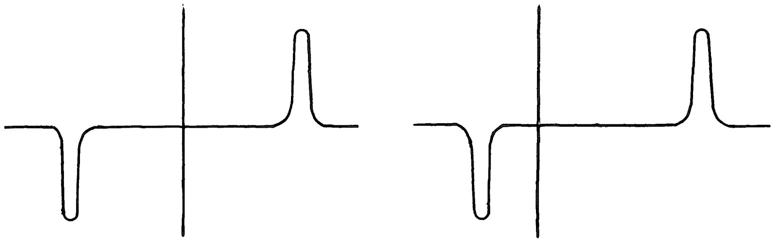
The hum is due to magnetostriction

Reference: Standard Handbook for Electrical Engineers, Sec 11-96 and following.

14-18 (14.9) POWER LOSS DUE TO HYSTERESIS

The area of the loop is approximately 2.8×16, or 45 W/m^3 cycle.

14-19 (14.9) THE FLUXGATE MAGNETOMETER AND THE PEAKING STRIP



Reference: H. Zijlstra, Experimental Methods in Magnetism, Vol 2, p 37, Brandt, Introduction to the Solar Wind p 145; M. Stanley Livingston and John P. Blewett, Particle Accelerators, p 276.

CHAPTER 15

15-1 (15.2) RELUCTANCE

$$W_m = (1/2)LI^2 = (1/2)(LI)I = (1/2)(N\Phi)I = (1/2)\Phi^2/NI = (1/2)\Phi^2\mathcal{R}$$

15-2 (15.2) RELUCTANCE

$$L = N\Phi/I = N^2\Phi/NI = N^2/\mathcal{R}$$

15-3 (15.3) CLIP-ON AMMETER

a) $\Phi = B_g A_g = \dfrac{I}{(2\pi R - L_g)/\mu_r\mu_o A + L_g/\mu_o A} \approx \mu_o IA/L_g$, $B \approx \mu_o I/L_g$

Without the iron core, B is $\mu_o I/2\pi R$ and is much smaller.

b) The position of the wire is unimportant.

15-4 (15.3) MAGNETIC CIRCUIT

For $\mu_r = 500$,

$$B = \frac{NI}{2\pi R/\mu_r\mu_o + \ell_g/\mu_o} = \frac{\mu_o NI}{2\pi R/\mu_r + \ell_g} = \frac{4\pi \times 10^{-7} \times 500 \times 2.4}{2\pi \times 0.2/500 + 10^{-3}} = \frac{1.508 \times 10^{-3}}{1.257/500 + 10^{-3}}$$

$$= 0.43 \text{ T.}$$

This B is too large; for $\mu_r = 500$, $B = 0.32$ T. Try $\mu_r = 525$

$$B = \frac{1.508\times 10^{-3}}{1.257/525+10^{-3}} = 0.44 \text{ T.}$$

This B is again too large; for $\mu_r = 525$, $B = 0.38$ T. Try $\mu_r = 550$

$B = 0.459$ T, instead of 0.5 on the graph. This is satisfactory.

15-5 (15.3) MAGNETORESISTANCE MULTIPLIER

$$V = \left[(R_2-R_1)/(R_2+R_1)\right] V = -\frac{4M^2B_aB_b}{2+2M^2\left(B_a^2+B_b^2\right)} V_0$$

$$= -\frac{2M^2B_aB_b}{1+M^2\left(B_a^2+B_b^2\right)} V_0 \approx -2M^2B_aB_bV_0$$

15-7 (15.4) RELAY

$$F = (B^2/2\mu_o)A = (A/2\mu_o)(\mu_oNI/\ell)^2 = (\mu_oA/2)(NI/\ell)^2$$

$$= (4\pi\times10^{-7}\times10^{-4}/2)(10^4\times10^{-2}/2\times10^{-3})^2 = 0.16 \text{ N.}$$

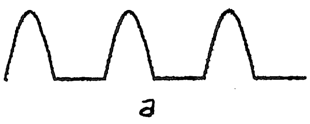
15-8 (15.3) MAGNETIC FLUIDS

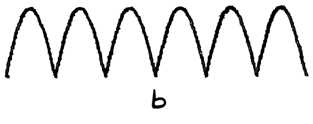
a) The magnetic flux is concentrated in the fluid, where B is large enough to give an appreciable magnetic pressure.

b) See the literature published by Ferrofluidics Corp.

CHAPTER 16

16-1 (16.1) RECTIFIER CIRCUITS COMPARED





16-2 (16.2) RMS VALUE OF A SINE WAVE

$$V_{rms}^2 = (1/T)\int_0^T V_0^2\cos^2\omega t\,dt = \left(V_0^2/\omega T\right)\int_0^{2\pi}\cos^2\alpha\,d\alpha = \left(V_0^2/2\pi\right)\pi,\ V_{rms} = V_0/2^{\frac{1}{2}}$$

16-4 (16.2) RMS VALUE

a) V_0, b) $\left(4V_0^2/2\right)^{\frac{1}{2}} = 2^{\frac{1}{2}}V_0$

c) $$\left\{(1/T)\int_0^T \left[V_0(1-2t/T)\right]^2 dt\right\}^{\frac{1}{2}} = (V_0/T^{\frac{1}{2}})\left[\int_0^T (1-2t/T)^2 dt\right]^{\frac{1}{2}}$$

$$= (V_0/T^{\frac{1}{2}})\left[T + \int_0^T (-4t/T)\,dt + \int_0^T (4t^2/T^2)\,dt\right]^{\frac{1}{2}} = (V_0/T^{\frac{1}{2}})\left[T-2T+(4/3)T\right]^{\frac{1}{2}}$$

$$= V_0/3^{\frac{1}{2}}$$

d) For the complete sine wave, $V_{rms} = V_0/2^{\frac{1}{2}}$, so that the mean square value is $V_0^2/2$. For the half sine wave, the mean square value is $V_0^2/4$, and $V_{rms} = V_0/2$.

16-5 (16.3) THREE-WIRE SINGLE-PHASE CURRENT

$2^{\frac{1}{2}}(240) = 339$ V

16-6 (16.4) THREE-PHASE CURRENT

$$\cos\omega t + \cos\omega t\cos(2\pi/3) - \sin\omega t\sin(2\pi/3) + \cos\omega t\cos(4\pi/3) - \sin\omega t\sin(4\pi/3)$$

$$= \cos\omega t[1+\cos(2\pi/3)+\cos(4\pi/3)] - \sin\omega t[\sin(2\pi/3)+\sin(4\pi/3)]$$

$$= \cos\omega t[1+\cos 120^\circ+\cos 240^\circ] - \sin\omega t[\sin 120^\circ+\sin 240^\circ]$$

$$= \cos\omega t[1+\cos 120^\circ+\cos(-120^\circ)] - \sin\omega t[\sin 120^\circ+\sin(-120^\circ)]$$

$$= \cos\omega t(1-0.5-0.5) - \sin\omega t(\sin 120^\circ-\sin 120^\circ) = 0$$

16-7 (16.4) ROTATING MAGNETIC FIELD

a) Using trigonometric functions,

$$B_x = B_o[\cos\omega t - 0.5\cos(\omega t+2\pi/3) - 0.5\cos(\omega t + 4\pi/3)]$$

$$= B_o[\cos\omega t - 0.5(\cos\omega t\cos 2\pi/3 - \sin\omega t\sin 2\pi/3)$$

$$-0.5(\cos\omega t\cos 4\pi/3 - \sin\omega t\sin 4\pi/3)]$$

$$= B_o[\cos\omega t - 0.5(-0.5\cos\omega t - 0.5\sqrt{3}\sin\omega t) - 0.5(-0.5\cos\omega t + 0.5\sqrt{3}\sin\omega t)]$$

$$= 1.5\ B_o\cos\omega t$$

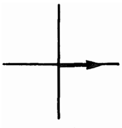
$B_y = B_o[0.5\sqrt{3}\cos(\omega t+2\pi/3) - 0.5\sqrt{3}\cos(\omega t+4\pi/3)]$

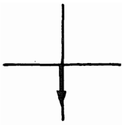
$= 0.5\sqrt{3}B_o(\cos\omega t\cos 2\pi/3 - \sin\omega t\sin 2\pi/3 - \cos\omega t\cos 4\pi/3+\sin\omega t\sin 4\pi/3)$

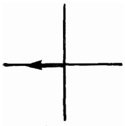
$= 0.5\sqrt{3}B_o(-0.5\cos\omega t - 0.5\sqrt{3}\sin\omega t + 0.5\cos\omega t - 0.5\sqrt{3}\sin\omega t)$

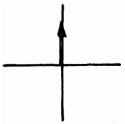
$= -2(0.5\sqrt{3})^2 B_o\sin\omega t = -1.5B_o\sin\omega t$

So $\vec{B} = 1.5B_o\cos\omega t\vec{i} - 1.5B_o\sin\omega t\vec{j}$, $B = 1.5\ B_o$.


$\omega t = 0$


$\omega t = \pi/2$


$\omega t = \pi$


$\omega t = 3\pi/2$

b) Using exponential functions,

$B_x = B_o[\exp j\omega t - 0.5\exp j(\omega t+2\pi/3) - 0.5\exp j(\omega t+4\pi/3)]$,

$= B_o\exp j\omega t(1-0.5\exp j2\pi/3-0.5\exp j4\pi/3)$,

$= B_o\exp j\omega t[1 - 0.5(-0.5+0.5\sqrt{3}j) - 0.5(-0.5-0.5\sqrt{3}j)]$,

$= 1.5B_o\exp j\omega t$,

$B_y = B_o[0.5\sqrt{3}\exp j(\omega t+2\pi/3) - 0.5\sqrt{3}\exp j(\omega t+4\pi/3)]$,

$= 0.5\sqrt{3}B_o\exp j\omega t(\exp j2\pi/3-\exp j4\pi/3)$,

$= 0.5\sqrt{3}B_o\exp j\omega t(-0.5+0.5\sqrt{3}j+0.5+0.5\sqrt{3}j)$,

$= 2(0.5\sqrt{3})^2 B_o j\exp j\omega t$,

$= 1.5B_o\exp j(\omega t+\pi/2)$.

Thus $\vec{B} = 1.5B_o(\cos\omega t\vec{i} - \sin\omega t\vec{j})$ as previously.

16-8 (16.4) DIRECT-CURRENT HIGH VOLTAGE TRANSMISSION LINES

a) $2(V_o/2^{\frac{1}{2}})I_{SP} = 2V_oI_{DC}$, $I_{SP} = 2^{\frac{1}{2}}I_{DC}$

b) $3(V_o/2^{\frac{1}{2}})I_{TP} = 2V_oI_{DC}$, $I_{TP} = (2/3)2^{\frac{1}{2}}I_{DC} = 0.94I_{DC}$

16-9 (16.5) ELECTROMAGNET OPERATING ON ALTERNATING CURRENT

$\Phi_{rms} = NI_{rms}/\mathcal{R} = (NV_{rms}/\omega L)/\mathcal{R} = V_{rms}/N\omega$, from Prob 15-2.

16-10 (16.6) COMPLEX NUMBERS

a) $1+2j = 2.236\angle 1.107$; $-1+2j = 2.236\angle 2.034$;

$-1-2j = 2.236\angle 4.249$; $1-2j = 2.236\angle -1.107$

b) $(1+2j)(1-2j) = 1+4 = 5$;

$$(1+2j)^2 = 1-4+4j = -3+4j$$

$$1/(1+2j)^2 = 1/(-3+4j) = (-3-4j)/(-3+4j)(-3-4j) = (-3-4j)/(9+16)$$

$$= -0.12 - 0.16j$$

$$(1+2j)/(1-2j) = (1+2j)(1+2j)/(1+4) = -0.6+0.8j$$

16-11 (16.6) COMPLEX NUMBERS

$2.236\angle 1.107$

16-12 (16.6) COMPLEX NUMBERS

$$\begin{aligned}
\exp jx &= 1 + jx - x^2/2! - jx^3/3! + x^4/4! + jx^5/5! \ \ldots \\
\cos x &= 1 \qquad\ \ - x^2/2! \qquad\qquad + x^4/4! \qquad\qquad\quad \ldots \\
j\sin x &= \quad + jx \qquad\qquad - jx^3/3! \qquad\qquad + jx^5/5! \ \ldots
\end{aligned}$$

16-13 (16.6) COMPLEX NUMBERS

$\exp j\pi = \cos\pi + j\sin\pi = -1$

16-14 (16.6) COMPLEX NUMBERS

a) Multiplication by j increases the argument θ by $\pi/2$ radians.

b) θ increases by π.

c) θ decreases by $\pi/2$.

16-15 (16.7) THREE-PHASE ALTERNATING CURRENT

$$\cos\omega t - \cos(\omega t+120^\circ) = \cos\omega t - \cos\omega t\cos 120^\circ + \sin\omega t\sin 120^\circ$$

$$= \cos\omega t(1+\cos 60^\circ) + \sin\omega t\sin 60^\circ = (3/2)\cos\omega t + (3^{\frac{1}{2}}/2)\sin\omega t$$

$$= 3^{\frac{1}{2}}\left[(3^{\frac{1}{2}}/2)\cos\omega t + (1/2)\sin\omega t\right] = (3^{\frac{1}{2}})(\cos 30^\circ\cos\omega t + \sin 30^\circ\sin\omega t)$$

$$= 3^{\frac{1}{2}}\cos(\omega t - 30^\circ) = 3^{\frac{1}{2}}\cos(\omega t - \pi/6)$$

16-16 (16.7) CALCULATING AN AVERAGE POWER WITH PHASORS

a) $$P_{av} = (1/T)\int_0^T V_o \cos\omega t I_o \cos(\omega t+\varphi)\,dt$$

$$= (V_o I_o/T)\int_0^T \cos\omega t(\cos\omega t\cos\varphi - \sin\omega t\sin\varphi)\,dt$$

$$= (V_o I_o/T)\left(\cos\varphi\int_0^T \cos^2\omega t\,dt - \sin\varphi\int_0^T \cos\omega t\sin\omega t\,dt\right) = (V_o I_o/T)\cos\varphi(T/2)$$

$$= (V_o I_o/2)\cos\varphi$$

The integral of $\sin\omega t\cos\omega t$ over one period is zero.

b) $$P_{av} = (1/2)R_c\left[V_o \exp(j\omega t) I_o \exp-j(\omega t+\varphi)\right] = (1/2)V_o I_o \cos\varphi$$

CHAPTER 17

17-1 (17.1) IMPEDANCE

a) $$Z = R + j\omega L + \frac{R/j\omega C}{R + 1/j\omega C} = R + j\omega L + \frac{R(1 - Rj\omega C)}{R^2\omega^2C^2 + 1}$$

$$= R + \frac{R}{R^2\omega^2C^2 + 1} + j\left(\omega L - \frac{R^2\omega C}{R^2\omega^2C^2 + 1}\right)$$

$Z = 2R$ at $f = 0$ and $Z \to j\omega L$ at $f \to \infty$

b) $R + R/(R^2\omega^2C^2+1) = 10 + 10/(100\text{x}4\pi^2\text{x}10^6\text{x}25\text{x}10^{-18}+1) = 20\Omega$

$\omega L - R^2\omega C/(R^2\omega^2C^2+1) = 2\pi\text{x}10^3\text{x}5\text{x}10^{-3} - 100\text{x}2\pi\text{x}10^3\text{x}5\text{x}10^{-9}/1 = 31.4\Omega$

$|Z| = (20^2+31.4^2)^{\frac{1}{2}} = 37.2\Omega$, $\varphi = \arctan(31.4/20) = 57.5^\circ$

c) $|Y| = 1/|Z| = 2.69 \times 10^{-2}\text{S}$, $\varphi = -57.5^\circ$

d) $P = 20I^2 = 0.20$ W

e) No

f) X is inductive (positive) for $L > R^2C/(R^2\omega^2C^2+1)$, which is always true

g) X is zero at $f = 0$.

17-2 (17.1) REAL INDUCTORS

$$Z = \frac{(R+j\omega L)/j\omega C}{R+j\omega L+1/j\omega C} = \frac{R+j\omega L}{Rj\omega C-\omega^2 LC+1} = \frac{(R+j\omega L)[(1-\omega^2 LC) - Rj\omega C]}{(1-\omega^2 LC)^2 + R^2\omega^2 C^2}$$

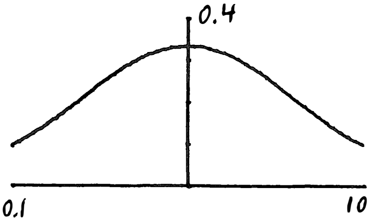
$$= \frac{R(1-\omega^2 LC) + R\omega^2 LC + j\omega[L(1-\omega^2 LC) - R^2 C]}{(1-\omega^2 LC)^2 + R^2\omega^2 C^2} = \frac{R+j\omega[L(1-\omega^2 LC) - R^2 C]}{(1-\omega^2 LC)^2 + R^2\omega^2 C^2}$$

17-3 (17.3) COMPENSATED VOLTAGE DIVIDER

$$V_o/V_i = \frac{\dfrac{R_2/j\omega C_2}{R_2 + 1/j\omega C_2}}{\dfrac{R_2/j\omega C_2}{R_2+1/j\omega C_2} + \dfrac{R_1/j\omega C_1}{R_1+1/j\omega C_1}} = \frac{\dfrac{R_2}{R_2 j\omega C_2 + 1}}{\dfrac{R_2}{R_2 j\omega C_2+1} + \dfrac{R_1}{R_1 j\omega C_1+1}}$$

This ratio is equal to $R_2/(R_1+R_2)$ if $R_1C_1 = R_2C_2$.

17-4 (17.3) RC FILTER



a) Call V' the potential at the connection between A and C :
$V_o/V' = D/(C+D)$,

$$V'/V_i = \frac{\dfrac{B(C+D)}{B+C+D}}{\dfrac{B(C+D)}{B+C+D} + A} = \frac{B(C+D)}{B(C+D) + A(B+C+D)},$$

$$V_o/V_i = (V_o/V')(V'/V_i) = \frac{BD}{B(C+D) + A(B+C+D)} = \frac{BD}{AB+(A+B)(C+D)}$$

b) $A = R$, $B = 1/j\omega C$, $C = 1/j\omega C$, $D = R$

$$V_o/V_i = \frac{R/j\omega C}{R/j\omega C + (R+1/j\omega C)^2} = \frac{R}{R + R^2 j\omega C + 1/j\omega C + 2R} = \frac{1}{3 +j(R\omega C - 1/R\omega C)}$$

$|V_o/V_i|$ is maximum at $R\omega C = 1$. Then $V_o/V_i = 1/3$.

When $|V_o/V_i| = (1/3)/2^{\frac{1}{2}}$, $R\omega C = 0.303$ or 3.30. The pass-band is very broad.

17-5 (17.3) MEASURING AN IMPEDANCE WITH A PHASE-SENSITIVE VOLTMETER

$$r = V_2/V_1 = Z/(R'+Z),$$

$$Z = R'r/(1-r) = R'(a+bj)/(1-a-bj)$$

$$\frac{Z}{R'} = \frac{(a+bj)(1-a+bj)}{(1-a)^2+b^2} = \frac{a(1-a)-b^2+jb)}{(1-a)^2+b^2}$$

Setting $Z = R + jX$,

$$\frac{R}{R'} = \frac{a-a^2-b^2}{1+a^2+b^2-2a} = \frac{1-|r|^2}{1+|r|^2-2a}, \quad \frac{X}{R'} = \frac{b}{1+|r|^2-2a}.$$

If $r = r_o\exp(j\theta) = r_o(\cos\theta + j\sin\theta)$, $a = r_o\cos\theta$, $b = r_o\sin\theta$,

$$\frac{R}{R'} = \frac{r_o\cos\theta - r_o^2}{1+r_o^2-2r_o\cos\theta}, \quad \frac{X}{R'} = \frac{r_o\sin\theta}{1+r_o^2-2r_o\cos\theta}$$

Reference : Electronics, July 25, 1974, p 117.

17-6 (17.3) IMPEDANCE BRIDGES. THE WIEN BRIDGE

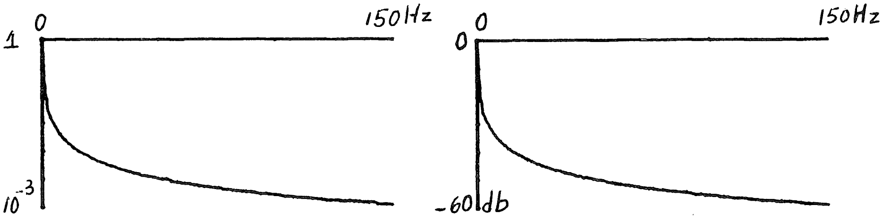
$$\frac{R_1}{R_2} = \frac{\dfrac{R_3/j\omega C_3}{R_3+1/j\omega C_3}}{R_4+\dfrac{1}{j\omega C_4}} = \frac{R_3}{(R_3 j\omega C_3+1)(R_4+1/j\omega C_4)}$$

$$= \frac{R_3}{\left[R_4+R_3(C_3/C_4)\right] + j(R_3R_4\omega C_3 - 1/\omega C_4)}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4 + R_3(C_3/C_4)}\ , \quad R_3R_4\omega^2C_3C_4 = 1$$

For $R_1 = R_2/2$, $R_3 = R_4$, $C_3 = C_4$, the first equation is satisfied and the second one yields $R_3\omega C_3 = 1$.

17-7 (17.3) LOW-PASS RC FILTER

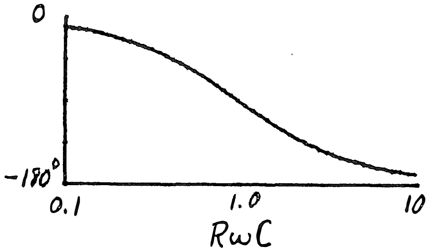


a) $V_o/V_i = \dfrac{1/j\omega C}{R + 1/j\omega C} = \dfrac{1}{Rj\omega C + 1}$, $|V_o/V_i| \approx 1/R\omega C$ if $R\omega C >> 1$.

b) $|V_o/V_i| = 1/(R^2\omega^2C^2+1)^{\frac{1}{2}} = 1/(10^8x4\pi^2xf^2x10^{-8}+1)^{\frac{1}{2}} = 1/(4\pi^2f^2+1)^{\frac{1}{2}}$

c) $db = 20\ \log|V_o/V_i|$.

17-8 (17.3) PHASE SHIFTER



$$V_o = \left(\frac{R}{R+1/j\omega C} - \frac{1/j\omega C}{R+1/j\omega C}\right)V_i = \frac{R-1/j\omega C}{R+1/j\omega C}\ V_i = \frac{R+j/\omega C}{R-j/\omega C}$$

$$V_o/V_i = \exp^{2j\ \mathrm{arc\ tg}(1/R\omega C)}$$

17-9 (17.3) MEASURING SURFACE POTENTIALS ...

a) $V = d_aE_a = d_dE_d$,

$$\sigma = \epsilon_oE_a + \epsilon_r\epsilon_oE_d = \epsilon_oE_a + \epsilon_r\epsilon_o(d_aE_a/d_d) = \epsilon_o(1+\epsilon_rd_a/d_d)E_a$$

$E_a = (\sigma/\epsilon_o)/(1+\epsilon_r d_a/d_d)$

b) $\sigma_i = \epsilon_o E_a = \sigma/(1+\epsilon_r d_a/d_d)$

c) $V = d_a E_a = (\sigma d_a/\epsilon_o)/(1+\epsilon_r d_a/d_d)$

d) $IR = (dQ/dt)R = V(dC/dt)R$

$C = (10^{-13}/2)(1+\exp j\omega t)$, $dC/dt = (10^{-13}/2)j\omega \exp j\omega t$

$IR = (10^{-13}/2)j\omega(\exp j\omega t)RV$

$I_{rms}R = (10^{-13}/2)(1/2^{\frac{1}{2}})\omega RV = (10^{-13}/2^{3/2})2\pi \times 100 \times 10^7 \times 10^3 = 0.224V$

References: Static Electrification 1975, pages 173, 182; Catalog of Monroe Electronics.

17-10 (17.3) REFERENCE TEMPERATURES NEAR ABSOLUTE ZERO

Let I_p and I_s be the currents in the primaries and in the secondaries, respectively. At balance, $V = 0$.

Then $j\omega M_1 I_p = R_1 I_s$, $j\omega M_2 I_p = R_2 I_s$, $M_1/M_2 = R_1/R_2$

Reference: Rev. Sci. Instr. 44, 1537 (1973).

17-11 (17.3) REMOTE-READING MERCURY THERMOMETER

a) Since $C_1 >> C_2$, the capacitance of C_1 and C_2 in series may be set equal to C_2 and

$$V'/V_s = \frac{R}{R + 1/j\omega C_2} = Rj\omega C_2/(Rj\omega C_2+1),$$

where C_2 varies linearly with the temperature.

b) Since V' is to vary linearly with C_2, we must have that $R\omega C_2 << 1$, and then $V' << V_s$

c) C_2 is a cylindrical capacitor with an outside radius of, say, 2 mm. The mercury column has a radius of, say, 0.05 mm. Then

$C_2 \approx 2\pi\epsilon_o\epsilon_r L/\ell n(2/0.05) = 2\pi\epsilon_o\epsilon_r L/\ell n\ 40,$

$\approx (2\pi x 8.85 x 10^{-12} x 3/\ell n\ 40)L \approx 45\ L$ pF.

We have set $\epsilon_r \approx 3$. Here L is the length of the mercury column inside the electrode C. Setting $L \approx 100$ mm, if $R_1 >> j\omega L_{1s}$, $R_2 >> j\omega L_{2s}$,

d) $C_2 \approx 5$ pF, $R << 1/\omega C_2 = 1/2\pi \times 10^7 \times 5 \times 10^{-12} = 3000\Omega$.

Setting $R \approx 30\Omega$,

$R\omega C_2 \approx 1/100$, $V'/V_s \approx 1/100$, $V' \approx 0.1$ volt.

Reference: Review of Scientific Instruments 47, 195 (1976).

17-12 (17.3) WATTMETER

Since $\omega L << Z$, we may set the voltage across the load equal to that at the source. Also, since $R_1 >> Z$, we may set the current through the load equal to the current supplied by the source.

The coil produces a B that is proportional to, and in phase with, the current through Z.

The voltage across R_2 is $R_2/(R_1+R_2)$ times the voltage across the source. Then

$V' = KI_o\cos(\omega t+\varphi)V_o\cos\omega t = KV_oI_o(\cos\omega t\cos\varphi - \sin\omega t\sin\varphi)\cos\omega t$

$V'_{av} = KV_oI_o(\cos\varphi/2) = KV_{rms}I_{rms}\cos\varphi$

17-13 (17.3) TRANSIENT SUPPRESSOR FOR AN INDUCTOR

We can calculate the voltage across the inductor, after the switch is opened, in another way. We consider a clockwise mesh current in L and C.

a) First, we find Q and I as functions of t, with the switch open. From Kirchoff's voltage law,

$LdI/dt + 2RI + Q/C = C$, $Ld^2Q/dt^2 + 2RdQ/dt + Q/C = 0$.

Try a solution of the form $Q = Q_o \exp nt$. Then $Ln^2 + 2Rn + 1/C = 0$

$n = -R/L \pm (R^2/L^2 - 1/LC) = -R/L$, since $R^2 = L/C$. The two roots are equal.

Then $Q = (A+Bt)\exp(-Rt/L)$.

Set $Q = Q_o$ at $t = 0$. Then $A = Q_o$.

Now $I = dQ/dt = \exp(-Rt/L)[-(R/L)(A+Bt)+B]$

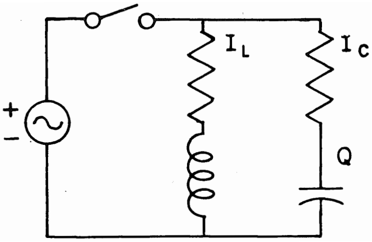
Set $I = I_o$ at $t = 0$. Then $B = I_o + RQ_o/L$,

$Q = \exp(-Rt/L)(Q_o + I_o t + RQ_o t/L)$,

$I = \exp(-Rt/L)[-(R/L)(Q_o + I_o t + RQ_o t/L) + I_o + RQ_o/L]$,

$= \exp(-Rt/L)[I_o - (R/L)(I_o + RQ_o/L)t]$.

b) Now let us find a relation between Q_o and I_o. Set I_L and I_C the currents through the inductor and capacitor before $t = 0$ in the directions shown, so as to give a clockwise current in the right-hand mesh when the switch is open. Then



$$I_L = -\frac{V_o}{(R^2+\omega^2L^2)^{\frac{1}{2}}}\cos[\,\omega t-\tan^{-1}(\omega L/R)],$$

$$I_C = \frac{V_o}{(R^2+1/\omega^2C^2)^{\frac{1}{2}}}\cos[\omega\, t+\tan^{-1}(1/R\omega C)]$$

$$= dQ/dt,$$

$$Q = \frac{V_o}{\omega(R^2+1/\omega^2C^2)^{\frac{1}{2}}}\sin[\omega\, t+\tan^{-1}(1/R\omega C)],$$

$$I_o = \frac{-V_o}{(R^2+\omega^2L^2)^{\frac{1}{2}}}\cos[-\tan^{-1}(\omega L/R)] = -V_oR/(R^2+\omega^2L^2),$$

$$Q_o = \frac{V_o}{\omega(R^2+1/\omega^2C^2)^{\frac{1}{2}}}\sin[\tan^{-1}(1/R\omega C)] = CV_o/(R^2\omega^2C^2+1)$$

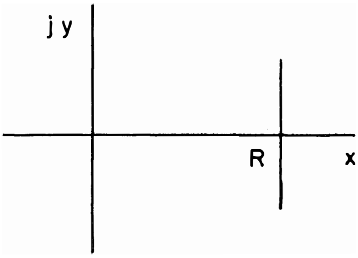
$$\frac{I_o}{Q_o} = -\frac{R}{C}\,\frac{(R^2\omega^2C^2+1)}{(R^2+\omega^2L^2)} = -R/L = -1/(LC)^{\frac{1}{2}}.$$

If the source supplied DC, instead of AC, I_o would be $-V/R$, Q_o would be VC, and I_o/Q_o would again be $-1/RC$, or $-1/(LC)^{\frac{1}{2}}$.
Since $I_o = -RQ_o/L$, $Q = Q_o\exp(-Rt/L)$, $I = I_o\exp(-Rt/L)$

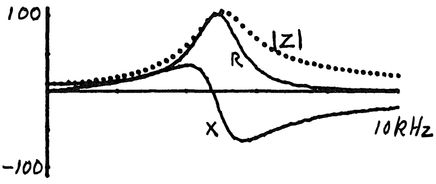
c) The voltage across the inductor, after the switch has been opened, is
$RI + LdI/dt = RI_o\exp(\ ) + L(-R/L)\exp(\ ) = 0$,
despite the fact that I decreases exponentially with time.
Reference: Reference Data for Radio Engineers, p 6-12.

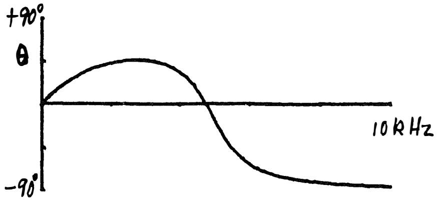
17-14 (17.3) SERIES RESONANCE

$Z = R + j(\omega L-1/\omega C)$
$Z \to -\infty\, j$ for $\omega\to 0$, $Z = R$ for $\omega^2LC = 1$,
$Z \to \infty j$ for $\omega \to \infty$



17-15 (17.3) PARALLEL RESONANCE





a) $Z = \frac{(R+j\omega L)/j\omega C}{R+j\omega L+1/j\omega C} = \frac{R+j\omega L}{Rj\omega C-\omega^2 LC+1}$

$= \frac{(R+j\omega L)(1-\omega^2 LC-Rj\omega C)}{(1-\omega^2 LC)^2+R^2\omega^2 C^2} = \frac{R+j\omega[L(1-\omega^2 LC)-R^2 C]}{(1-\omega^2 LC)^2+R^2\omega^2 C^2}$

Real part of $Z = \frac{R}{(1-\omega^2 LC)^2+(R^2\omega^2 C^2)} = \frac{10}{(1-4\pi^2 f^2 10^{-9})^2+10^{-10}4\pi^2 f^2}\ \Omega$

Imaginary part of $Z = \frac{2\pi f[10^{-3}(1-4\pi^2 f^2 10^{-9})-10^{-4}]}{(\quad)^2+10^{-10}4\pi^2 f^2}\ \Omega$

Magnitude of $Z = \frac{\{100+4\pi^2 f^2[\quad]^2\}^{\frac{1}{2}}}{(\quad)^2+10^{-10}4\pi^2 f^2}\ \Omega$

Phase of $Z = \arctan \omega[\quad]/R$

b) $X = 0$ when $[\quad] = 0$, or when

$L(1-\omega^2 LC) = R^2 C,\ 1-\omega^2 LC = R^2 C/L,\ \omega^2 = (1-R^2 C/L)/LC$

$f = [(1-R^2 C/L)/LC]^{\frac{1}{2}}/2\pi = [(1-10^{-4}/10^{-3})/10^{-9}]^{\frac{1}{2}}/2\pi = 4.77$ kHz

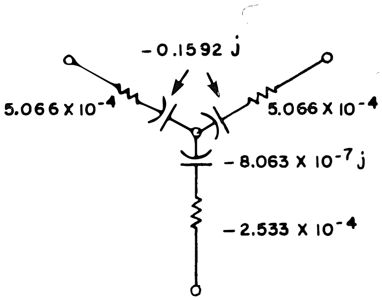
c) $1-\omega^2 LC = 1-4\pi^2 \times 64\times 10^6\times 10^{-3}\times 10^{-6} = 1-4\pi^2\times 64\times 10^{-3} = -1.527$

$Z = \frac{10+j(2\pi\times 8\times 10^3)[10^{-3}(-1.527)-10^2\times 10^{-6}]}{2.332+10^2\times 4\pi^2\times 64\times 10^6\times 10^{-12}} = \frac{10-81.78j}{2.584}$

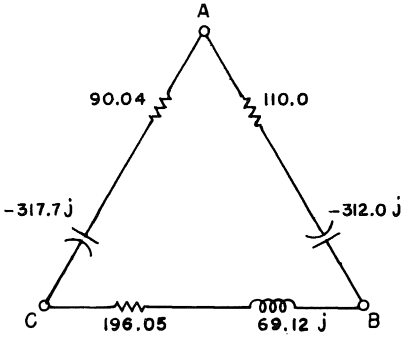
d) $Z = R' + 1/j\omega C'$, $R' = 3.869$, $C' = 0.629$ μF

Reference: Philips Technical Review 31 No 4 (1971).

17-16 (17.4) STAR-DELTA TRANSFORMATION



17-17 (17.4) STAR-DELTA TRANSFORMATION



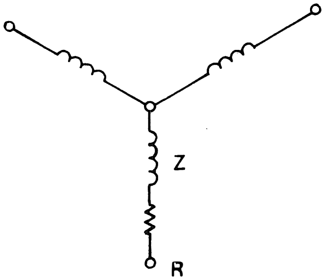
17-19 (17.4) BRIDGED-T

$R + Z = 0$, $R = -Z$,

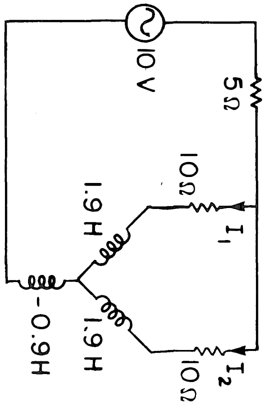
$1/R = -Y = -[2j\omega C - \omega^2 C^2 (r + j\omega L)]$

$= -2j\omega C + \omega^2 C^2 r + j\omega^3 LC^2$

Then $\omega^2 C^2 rR = 1$, $\omega^2 LC = 2$.



17-20 (17.5) MUTUAL INDUCTANCE



Transform the circuit into the one shown in the figure of the preceding page.

$$I_1 = I_2 = (\tfrac{1}{2})\left|\frac{10}{5+\frac{10+2\pi\times10^3\times1.9j}{2}-2\pi\times10^3\times0.9j}\right|$$

$$= 5/|5+5+100\pi j| = 5/|10+314j| = 5/(10^2+314^2)^{\frac{1}{2}}$$

$$= 1.592\times10^{-2}\text{A} = 15.92 \text{ mA}$$

CHAPTER 18

18-1 (18.1) DIRECT-CURRENT MOTORS

a) Replace V' by a resistance $R' = V'/I$. Then the power supplied by the source is

$IV = I(IR+IR') = I^2R + IV' = I^2R + I^2R'$.

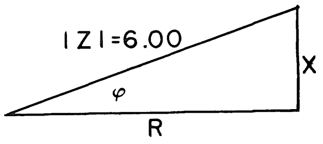
The first term represents the various losses and IV' is the useful power.

b) $\mathcal{E} = I^2R'/(I^2R+I^2R') = R'/(R+R')$

c) At no load, $V' \to V$, $I = (V-V')/R \to 0$, $B \to 0$.
Since $V \approx V' \propto \omega B$, $\omega \to \infty$.

d) IV' increases. The motor slows down and V' decreases, so I increases faster, $R' = V'/I$ decreases and the efficiency decreases.

18-2 (18.2) POWER-FACTOR CORRECTION



a) $|Z| = 600/100 = 6.00\ \Omega$
$R = 6\times0.65 = 3.90\ \Omega$,
$X = 6\times\sin(\arccos 0.65) = 4.55\ \Omega$

b) $I = V/Z = V(R-jX)/|Z|^2$
$= (600/36)(3.90-j\,4.55)$

The in-phase component is
$(600/36)3.90 = 65.0$ A.

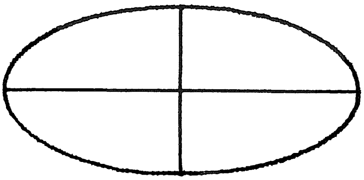
The quadrature component is 76.0 A, lagging. Check: $65.0^2 + 76.0^2 = 100^2$.

c) $V\omega C = 76$, $C = 76/(600 \times 2\pi \times 60) = 366\ \mu F$.
This capacitor would cost about $ 400.00.
Reference: Standard Handbook for Electrical Engineers, 5-98 and 16-185.

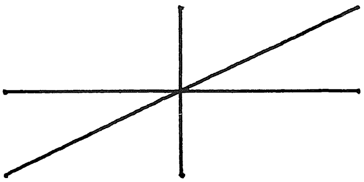
18-3 (18.2) POWER-FACTOR CORRECTION WITH FLUORESCENT LAMPS

The in-phase component of the current is 80/120 = 0.667 A.
Since $\cos\varphi = 0.5$, $\varphi = 60$ degrees. The current is $2 \times 0.667 = 1.33$ A
The reactive current is $1.33 \sin 60^{\circ} = 1.16$ A.
Then $V\omega C = 1.16$, $C = 1.16/(2\pi \times 60 \times 120) \approx 20\ \mu F$.
References: Henderson and Marsden, Lamps and Lighting, p 325
Standard Handbook for Electrical Engineers, 19-33.

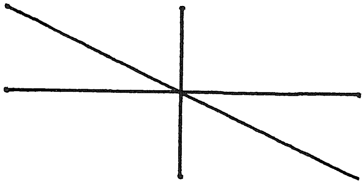
18-4 (18.3) ENERGY TRANSFER TO A LOAD



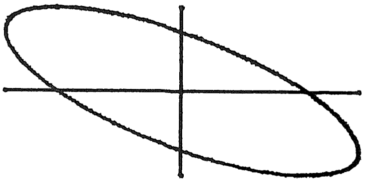
R



L



C



R(1+j)

a) $rI = RI' + (Q'/C)$, $rj\omega Q = Rj\omega Q' + Q'/C \approx Rj\omega Q'$
$Q' \approx (r/R)Q$, $x \propto (r/RC)Q$

b) $$W = \int_0^T VI\,dt = \int_{\text{one cycle}} V\,dQ \propto \int_{\text{one cycle}} y\,dx$$

c) i) For a resistor, $V = V_o \cos\omega t$, $I = (V_o/R)\cos\omega t = dQ/dt$,
$Q = (V_o/\omega R)\sin\omega t$.
See Fig. R

ii) For a capacitor, $V = V_o\cos\omega t$, $Q = CV_o\cos\omega t$.
See Fig. C.

iii) For an inductor, $V = V_o\cos\omega t$, $L\,dI/dt = V_o\cos\omega t$,
$L\,d^2Q/dt^2 = V_o\cos\omega t$, $Q = -(V_o/\omega^2 L)\cos\omega t$.
See Fig. L.

iv) For a resistor in series with an inductor, with $R = j\omega L$,

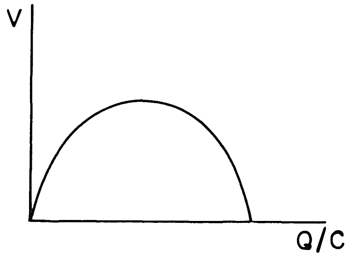
$$j\omega Q = I = V_o \exp j\omega t/(R+j\omega L) = (V_o/R)\exp j\omega t/(1+j)$$
$$= (V_o/2^{\frac{1}{2}}R)\exp j(\omega t-\pi/4),$$
$$Q = (V_o/2^{\frac{1}{2}}\omega R)\exp j(\omega t-\pi/4-\pi/2)$$

See the fourth figure.

Reference: Rev. Sci. Instr. 42, 109 (1971).

18-5 (18.3) ENERGY TRANSFER TO A LOAD

Let the voltage across G, at a given instant, be V. The voltage at y is then approximately equal to V. Let the current through Z, at a given instant, be I, and the pulse duration be T. Then the energy dissipated in Z during a pulse is



$$W = \int_0^T VI\,dt = \int_{\text{one cycle}} V\,dQ$$

The voltage at x is Q/C. Then the spot on the oscilloscope screen describes a curve as in the figure.
The area under this curve is proportional to the above integral.

Reference: Rev. Sci. Instr. 45, 1004 (1974).

18-6 (18.4) REFLECTED IMPEDANCE

$$Z_{in} = R_1 + jX_1 + \omega^2M^2/\left(R_2 + jX_2\right) = R_1 + jX_1 + \omega^2M^2(R_2-jX_2)/\left(R_2^{\ 2} + X_2^{\ 2}\right)$$
$$= \left[R_1 + \omega^2M^2R_2/\left(R_2^{\ 2}+X_2^{\ 2}\right)\right] + j\left[X_1 - \omega^2M^2X_2/\left(R_2^{\ 2}+X_2^{\ 2}\right)\right]$$

A positive X_2 is equivalent to a negative X in the primary.

18-7 (18.4) MEASUREMENT OF THE COEFFICIENT OF COUPLING k

With the secondary open, $Z_\infty = Z_1 = j\omega L_1$.

With the secondary short-circuited,

$$Z_0 = j\omega L_1 + \omega^2 M^2 / j\omega L_2 = j\omega L_1 - j\omega L_1 j\omega L_2 k^2 / j\omega L_2 = j\omega L_1 (1-k^2),$$

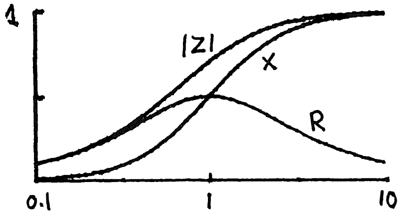
$$Z_0 / Z_\infty = 1 - k^2.$$

18-8 (18.4) REFLECTED IMPEDANCE

a) $$Z_{in} = R_1 + j\omega L_1 + \omega^2 M^2 / (R_2 + j\omega L_2) = j + 1/(R_2 + j) = j + (R_2 - j) / \left(R_2^2 + 1\right)$$

$$= (1 + jR_2) R_2 / \left(R_2^2 + 1\right)$$

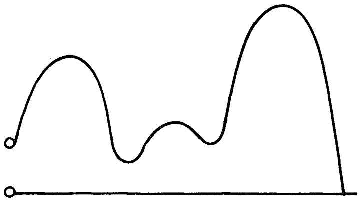
b)



18-9 (18.4) MEASURING THE AREA UNDER A CURVE

Draw a line around the periphery of the figure with conducting ink. Then measure the voltage induced when the Helmholtz coils are fed, say at 1 kilohertz.

The system can be calibrated with a circle or with a rectangle of known area.



Reference: Rev. Sci. Instr. 41, 1663 (1970).

18-10 (18.4) SOLDERING GUNS

a) $R = \ell / A\sigma = 10^{-1} / 4 \times 10^{-6} \times 5.8 \times 10^{7} = 4.3 \times 10^{-4}\ \Omega$

$I_{sec} = (100 / 4.3 \times 10^{-4})^{\frac{1}{2}} = 480$ A, $V_{sec} = 480 \times 4.3 \times 10^{-4} \approx 0.2$ volt

b) $I_{pri} = 100/120 = 0.8$ A.

18-11 (18.4) CURRENT TRANSFORMER

Disregarding the sign, the induced electromotance is $d\Phi/dt$, with

$$\Phi = \int_{b-a}^{b+a} (\mu_o I/2\pi r)2a\,dr = (\mu_o a/\pi)I\ell n[(b+a)/(b-a)],$$

$$V = (\mu_o a/\pi)\ell n[(b+a)/(b-a)]dI/dt.$$

Reference: Rev. Sci. Instr. 46, 324 (1975).

18-12(18.4) INDUCED CURRENTS

$R_t = \ell/\sigma A = 2\pi a/\ell b\sigma$

The tube is a single-turn solenoid. Hence $L = \mu_o \pi a^2/\ell$.

18-13 (18.5) EDDY-CURRENT LOSSES

See the standard Handbook for Electrical Engineers, Sec. 2-74.

18-14 (18.5) EDDY-CURRENT LOSSES

For a solid core, the power loss is

$$P_1 = V^2/R \approx (d\Phi/dt)^2/(4a/\sigma bL) = (\sigma bL/4a)(d\Phi/dt)^2.$$

If the core is split into n laminations, insulated one from the other,

$$P_n = n[d(\Phi/n)/dt]^2/(4an/\sigma bL) = P_1/n^2.$$

Reference: Standard Handbook for Electrical Engineers, Sec. 2-74, 91, 92, 93 and following.

18-15 (18.5) HYSTERESIS LOSSES

Place the laminations inside a solenoid and measure the resistive part R of the impedance of the solenoid as a function of the frequency.

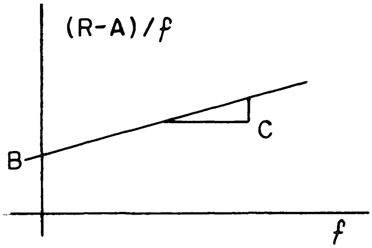
Then

$$R = A + Bf + Cf^2,$$

where A is the DC resistance.

Then

$$(R-A)/f = B + Cf.$$



A plot of (R-A)/f as a function of f gives both B and C.

18-16 (18.5) CLIP-ON AMMETER

a) $V = NAdB/dt = NA\omega B = NA\omega\mu_r\mu_o I/2\pi r$

$= 10^3(0.64 \times 10^{-4})(2\pi \times 60)(4\pi \times 10^{-7} \times 10^4)/2\pi \times 1.5 \times 10^{-2} = 3.12$ V

b) Loop the wire carrying the unknown current several times around the core.

CHAPTER 19

19-2 (19.3) MAXWELL'S EQUATIONS

$\epsilon_o\nabla\cdot\vec{E} = \rho_f - \nabla\cdot\vec{P}$, $\nabla\cdot(\epsilon_o\vec{E}+\vec{P}) = \rho_f$, $\nabla\cdot\vec{D} = \rho_f$, from Sec 6.5

$\nabla \times \vec{B} - \epsilon_o\mu_o\partial\vec{E}/\partial t = \mu_o(\vec{J}_f + \partial P/\partial t + \nabla \times \vec{M})$.

$\mu_o\nabla \times (\vec{H} + \vec{M}) - \epsilon_o\mu_o\partial\vec{E}/\partial t = \mu_o(\vec{J}_f + \partial\vec{P}/\partial t + \nabla \times \vec{M})$, from Eq. 14-20.

Dividing by μ_o and canceling $\nabla \times \vec{M}$ on both sides,

$\nabla \times \vec{M} = \vec{J}_f + (\partial/\partial t)(\epsilon_o\vec{E} + \vec{P}) = \vec{J}_f + \partial\vec{D}/\partial t$, from Eq. 6-5

19-3 (19.3) MAXWELL'S EQUATIONS

Use the equations of the previous problems, setting $\vec{D} = \epsilon_r\epsilon_o\vec{E}$, $\vec{B} = \mu_r\mu_o\vec{H}$, $\partial/\partial t = j\omega$

19-4 (19.3) MAXWELL'S EQUATIONS

$\nabla \times \vec{B} - \epsilon_o\mu_o\partial\vec{E}/\partial t = \mu_o\vec{J}_m$

Taking the divergence of both sides and remembering that $\nabla\cdot\nabla \times \vec{B} = 0$ for any vector $\vec{B}$,

$-\epsilon_o\mu_o(\partial/\partial t)(\nabla\cdot\vec{E}) = \mu_o\nabla\cdot(\vec{J}_f + \partial\vec{P}/\partial t)$.

Since $\nabla\cdot\vec{E} = (\rho_f + \rho_b)/\epsilon_o$,

$-(\partial/\partial t)(\rho_f + \rho_b) = \nabla\cdot\vec{J}_f + (\partial/\partial t)(\nabla\cdot\vec{P})$.

But $\rho_b = -\nabla\cdot\vec{P}$ and $\partial\rho_f/\partial t = -\nabla\cdot\vec{J}_f$.

19-5 (19.3) MAGNETIC MONOPOLES AND MAXWELL'S EQUATIONS

a) Taking the divergence of the equation for $\nabla \times \vec{E}$ and remembering that the divergence of a curl is always equal to zero,

$$(\partial/\partial t)(\nabla\cdot\vec{B}) = -\nabla\cdot\vec{J}^*, \quad \nabla\cdot\vec{J}^* = -\partial\rho^*/\partial t.$$

b) From the equation for the curl of $\vec{E}$,

$$\int_S \nabla\times\vec{E}\cdot d\vec{a} = \oint_C \vec{E}\cdot d\vec{\ell} = -\int \vec{J}^*\cdot d\vec{a} = -I^*.$$

19-6 (19.3)

$1\text{J} = 1\text{N.m} = 1\ \text{kg}(\text{m/s}^2)\text{m} = 1\ \text{kg m}^2/\text{s}^2$

$1\text{W} = 1\text{J/s} = 1\ \text{kg m}^2/\text{s}^3$

$1\text{C} = 1\text{As}$

$1\text{V} = 1\text{J/C} = 1(\text{kg m}^2/\text{s}^2)/\text{As} = 1\ \text{kg m}^2/\text{As}^3$

$1\Omega = 1\text{V/A} = 1\ \text{kg m}^2/\text{A}^2\text{s}^3$

$1\text{S} = 1\Omega^{-1} = 1\text{A}^2\text{s}^3/\text{kg m}^2$

$1\text{F} = 1\text{C/V} = 1\text{As}/(\text{kg m}^2/\text{As}^3) = 1\text{A}^2\text{s}^4/\text{kg m}^2$

$1\text{Wb} = 1\text{Vs}^* = 1\ \text{kg m}^2/\text{As}^2$

$1\text{T} = 1\text{Wb/m}^2 = 1\ \text{kg/As}^2$

$1\text{H} = 1\text{Wb/A}^{**} = 1\ \text{kg m}^2/\text{A}^2\text{s}^2$

*From the fact that, in a changing magnetic field, the induced voltage is equal to the rate of change of the magnetic flux

**From $L = \Phi/I$

Let us check

a) $j\omega LI = V$ gives $\frac{1}{\text{s}}\frac{\text{kg m}^2}{\text{A}^2\ \text{s}^2}\text{A} = \frac{\text{kg m}^2}{\text{A s}^3}$, Correct

b) The energy stored in a capacitor is $CV^2/2$. Then

$$\frac{\text{kg m}^2}{\text{s}^2} = \frac{\text{A}^2\text{s}^4}{\text{kg m}^2}\left(\frac{\text{kg m}^2}{\text{A s}^3}\right)^2. \quad \text{Correct}$$

c) The energy stored in an inductor is $LI^2/2$. Then

$$\frac{\text{kg m}^2}{\text{s}^2} = \frac{\text{kg m}^2}{\text{A}^2\text{ s}^2}\text{A}^2. \quad \text{Correct}$$

d) The power loss in a resistor is I^2R. Then

$$\frac{\text{kg m}^2}{\text{s}^3} = \text{A}^2\,\frac{\text{kg m}^2}{\text{A}^2\text{ s}^3}\,. \quad \text{Correct}$$

e) $\omega^2 LC$ is a pure number. Then

$$\frac{1}{\text{s}^2}\,\frac{\text{kg m}^2}{\text{A}^2\text{ s}^2}\,\frac{\text{A}^2\text{ s}^4}{\text{kg m}^2} = 1. \quad \text{Correct}$$

etc.

CHAPTER 20

20-1 (20.4) PLANE WAVE IN FREE SPACE

$E = E_o \exp j(\omega t - z/\bar{\lambda})$, $H = H_o \exp j(\omega t - z/\bar{\lambda})$,

where E_o and H_o are independent of x, y, z, t and have no z-component

a) Then, from $\nabla\cdot E = 0$,

$$(\partial/\partial x)\left[E_{ox}\exp j(\quad)\right] = 0, \quad (\partial/\partial y)\left[E_{oy}\exp j(\quad)\right] = 0$$

These equations are identities

b) We have similar equations for H.

c) From $\nabla \times \vec{E} = -\mu_o\,\partial\vec{H}/\partial t$,

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \partial/\partial z \\ E_x & E_y & 0 \end{vmatrix} = -\mu_o\,\partial\vec{H}/\partial t, \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -j/\bar{\lambda} \\ E_x & E_y & 0 \end{vmatrix} = -j\omega\mu_o\vec{H}$$

Thus, $(-j/\bar{\lambda})\vec{k}\times\vec{E} = -j\omega\mu\vec{H}$, $\vec{k}\times\vec{E} = \mu_o\omega\bar{\lambda}\vec{H} = \mu_o c\vec{H}$

d) From $\nabla \times \vec{H} = \epsilon_o\,\partial\vec{E}/\partial t$, $\vec{k}\times\vec{H} = -\epsilon_o c\vec{E}$

20-2 (20.4) LOOP ANTENNA

$$\mathcal{V}_{max} = 10(d\Phi/dt)_{max} = 10\cos 30^o(dB/dt)_{max} = 10\cos 30^o(dE/dt)_{max}/c$$

$$= 10\cos 30^o(2\pi\times 3\times 10^6)0.1/3\times 10^8 = 54.4\text{ mV}.$$

20-3 (20.6) POYNTING VECTOR

a) $\mathcal{S} = 3.8 \times 10^{26}/4\pi \times 49 \times 10^{16} = 2.65 \times 10^{-3} E^2_{rms}$, $E_{rms} = 1.53 \times 10^5$ V/m

b) $\mathcal{S}_E/\mathcal{S}_S = E_E^2/E_S^2 = (1/R_{S-E})^2/(1/R_s)^2$, $E_E/E_S = R_S/R_{S-E}$

$E_E = (7 \times 10^8/1.5 \times 10^{11})1.53 \times 10^5 = 700$ V/m

c) $\mathcal{S}_E = 2.65 \times 10^{-3} \times (700)^2 = 1.3 \times 10^3$ W/m^2

$= 60 \times 1.3 \times 10^3 (\text{cal}/4.19)/(10^4 \text{ cm}^2) = 1.86$ calorie/minute centimeter^2

This quantity is called the solar constant

We have neglected absorption in the atmosphere.

The average daily flux at the ground, in the United States, is about 0.4 calorie/minute centimeter^2.

Reference: American Institute of Physics Handbook, 3rd ed, p 2-143.

20-4 (20.6) SOLAR ENERGY

At the surface of the earth, $\mathcal{S} = 1.3 \times 10^3$ W/m^2, from Prob. 20-3.

$P = \mathcal{S}A/50$, $A = 50\,P/\mathcal{S} = 5 \times 10^7/1.3 \times 10^3 \approx 4 \times 10^4$ m^2,

or a square 200 meters on the side.

20-5 (20.6) POYNTING VECTOR

$\mathcal{S} = c\epsilon_o E^2 = 3 \times 10^8 \times 8.85 \times 10^{-12} \times 20^2 = 1.06$ W/m^2

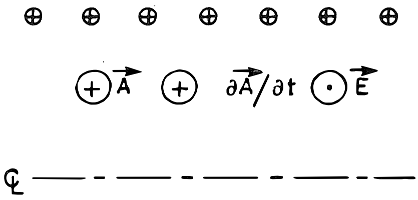
In one second, the energy absorbed by one square meter of the copper sheet is 1.06 J.

This energy will increase the temperature of one kilogram of copper by 1.06/400 kelvin.

In one second the temperature of the sheet rises by 100 x 1.06/400 = 1.06/4 = 0.265 kelvin.

20-6 (20.6) POYNTING VECTOR

a)



b) Energy flows into the field.

20-7 (20.6) POYNTING VECTOR

$\vec{E}_1 = E_{10}\exp(\omega t - kz)\vec{i}$, $\vec{E}_2 = E_{20}\exp(\omega t - kz - \pi/2)\vec{j}$

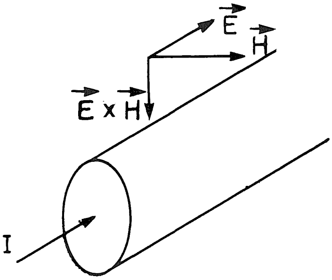
$\vec{H}_1 = H_{10}\exp(\omega t - kz)\vec{j}$, $\vec{H}_2 = H_{20}\exp(\omega t - kz - \pi/2)(-\vec{i})$

$\vec{\mathcal{S}}_{av} = (1/2)\mathrm{Re}\{(\vec{E}_1 + \vec{E}_2) \times (\vec{H}_1 + \vec{H}_2)^*\} = (1/2)\mathrm{Re}\ \vec{E}_1 \times \vec{H}_1^* + (1/2)\mathrm{Re}\ \vec{E}_2 \times \vec{H}_2^*$

20-8 (20.6) POYNTING VECTOR

Near the surface of the wire,
$E = IR'$, $H = I/2\pi a$, $|\vec{E}\times\vec{H}|$
$= I^2R'/2\pi a$

Thus the power loss per meter is I^2R'.



20-10 (20.6) COAXIAL LINE

$E = \lambda/2\pi\epsilon_o r$, $\int_{R_1}^{R_2} (\lambda/2\pi\epsilon_o r)dr = V$, $(\lambda/2\pi\epsilon_o)\ell n(R_2/R_1) = V$,

$\lambda = 2\pi\epsilon_o V/\ell n(R_2/R_1)$, $E = V/r\ell n(R_2/R_1)$

$H = I/2\pi r$

$$\int_{R_1}^{R_2} [V/r\ell n(R_2/R_1)](I/2\pi r)2\pi r dr = VI = 220 \times 10 = 2200 \text{ W}$$

20-11 (20.7) REFLECTION AND REFRACTION, FRESNEL'S EQUATIONS

See Electromagnetic Fields and Waves, Sec 12.2.2.