Physica Scripta

Exploring the dynamic interplay of intermodal and higher order dispersion in nonlinear negative index metamaterials

To cite this article: S Saravana Veni *et al* 2024 *Phys. Scr.* **99** 085261

IOP Publishing *Phys. Scr.* **99** (2024) 085261 https://doi.org/10.1088/1402-4896/ad6352

Physica Scripta

RECEIVED
20 February 2024

REVISED
22 June 2024

ACCEPTED FOR PUBLICATION
15 July 2024

PUBLISHED
25 July 2024

PAPER

Exploring the dynamic interplay of intermodal and higher order dispersion in nonlinear negative index metamaterials

S Saravana Veni[1], M S Mani Rajan[2], Anjan Biswas[3,4,5,6] and Ali Saleh Alshomrani[6]

[1] Department of Physics, Amrita School of Physical Sciences, Amrita Vishwa Vidyapeetham, Coimbatore, Tamilnadu, India
[2] Department of Physics, University College of Engineering, Anna University, Ramanathapuram, India
[3] Department of Mathematics and Physics, Grambling State University, Grambling, LA 71245–2715, United States of America
[4] Department of Applied Sciences, Cross–Border Faculty of Humanities, Economics and Engineering, Dunarea de Jos University of Galati, 111 Domneasca Street, Galati, 800201, Romania
[5] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa–0204, Pretoria, South Africa
[6] Mathematical Modeling and Applied Computation (MMAC) Research Group, Center of Modern Mathematical Sciences and their Applications (CMMSA), Department of Mathematics, King Abdulaziz University, Jeddah, 21589, Saudi Arabia

E-mail: s_saravanaveni@cb.amrita.edu

Keywords: meta materials, higher order nonlinear schrodinger wave equation, modulation instability, numerical analysis

Abstract

Our study delves into the intricate interplay of various factors within metamaterials, with a focus on modulation instability. Through our research, we elucidate the intricate dynamics involving intermodal dispersion, self-steepening effect, higher order dispersion, and plane wave amplitude, showcasing their competition and influence on modulation instability phenomena. We aim to explore the impact of intermodal dispersion and higher-order effects by numerically solving the generalized nonlinear Schrödinger equation (NLSE), which models the propagation of a few-cycle pulse in a nonlinear metamaterial. Our modulation instability (MI) analysis captures the complex dynamics these factors introduce. We demonstrate the spatiotemporal evolution of MI under different parameter values, revealing how these variations influence the instability's development and characteristics. This approach provides a detailed understanding of the system's behavior across various conditions, highlighting the roles of intermodal dispersion and higher-order effects. We demonstrate that the behavior of modulation instability bands and their reliance on parameters such as self-steepening and wave amplitude is contingent upon the specific characteristics of the optical setup and medium dispersion properties

1. Introduction

Cross-phase modulation-induced modulation instability is a phenomenon that occurs in nonlinear optics, specifically in the context of optical fiber communication systems. It is a complex and mathematically intensive topic that involves the interaction of optical pulses within a fiber optic medium. A realistic experimental example would involve a controlled optical fiber setup to observe and measure the effects of these interactions on the optical pulses. The quantitative aspects of this cross-phase–modulation–induced modulation instability is discussed and illustrated by use of a realistic experimental example [1]. As two optical beams co-propagate within a single-mode fiber, their interaction occurs via the intensity-dependent refractive index, a nonlinear effect known as cross-phase modulation (XPM). This interaction, induced by XPM, has the capacity to perturb the steady state, resulting in temporal modulations (self-pulsing), particularly when influenced by group-velocity dispersion. Remarkably, XPM-induced modulation instability is not limited to the normal-dispersion regime. A comprehensive analysis is presented on the attributes of XPM-induced modulation instability. It is worth noting that this phenomenon offers the possibility of generating ultrashort pulses ($<100fs$) in the optical spectrum's visible domain [2]. An experimental study on the modulation instability process and the associated rogue breathers is being reported for the case of stationary periodic background waves, specifically, dnoidal and cnoidal envelopes. In the domains of nonlinear optics and hydrodynamics, the spontaneous modulation instability gain seeded by input random noise and the formation of rogue breathers induced by a coherent perturbation are observed by means of two experimental setups [3]. The space-time evolution of a modulationally unstable plane wave initially perturbed by a small noise is examined, and the noise-driven development of breather structures from the early stage to the long-term evolution of modulation instability is reported, using a recirculating fiber loop as the experimental platform [4, 5]. The discovery of an important category of 'superregular solitonic solutions' is made, where they are characterized as small perturbations occurring at a particular moment in time. The nonlinear stage of the modulation instability of the condensate is elucidated by these solutions [6]. The nonlinear magnetoinductive waveguide for microwaves is fabricated using varactor-loaded split-ring resonators, and the generation of modulation instability in the waveguide is observed. The condition for generating modulation instability in the experiment is roughly in agreement with that in the numerical analysis [7]. The spatial nonlinear localization of light on a quasi-plane-wave background with a harmonic perturbation induced by modulation instability in a quadratic nonlinear optical medium is investigated. In particular, the excitation of deterministic Akhmediev breathers and the growth-decay dynamics of modulation instability in a LiNbO3 slab waveguide are demonstrated experimentally [8]. The strong modulation of frequency within the bursts of unstable waves is attributed to the presence of propagating strongly nonlinear coherent structures that emerge during the nonlinear spatiotemporal evolution of the ion-acoustic instability, giving rise to the observed broad power spectra of the density fluctuations [9]. To investigate the self-modulation of the proton bunch and its transition to the nonlinear stage, comprehensive three-dimensional particle-in-cell (PIC) simulations are utilized. A competition is revealed in these simulations between the self-modulation of the proton bunch and the hosing instability, which exerts a destructive influence on the plasma wave by considering an analytical model for the self-modulation instability of a long relativistic proton bunch propagating in uniform plasmas [10]. Experiments were conducted in a coherently driven nonlinear optical resonator, and they demonstrated the nonlinear localization of dissipative modulation instability (MI). This involved the formation of persisting domains of MI-driven spatiotemporal chaos, which were surrounded by a stable quasi-plane-wave background. The persisting localization was brought about by a combination of bistability and complex spatiotemporal nonlinear dynamics, which together allowed a locally induced domain of MI to be pinned by a shallow modulation on the plane wave background [11]. It is demonstrated how higher-order modulation instability splitting is induced by a suitably low-frequency modulation on a continuous wave field, with the pulse characteristics at different phases of evolution being related by a simple scaling relationship [12]. The effect of external stochastic modulation on a system with O(2) symmetry that exhibits a Hopf or oscillatory instability in the absence of modulation is being studied with the inclusion of random component in both the control parameter of the bifurcation and the modulation amplitude [13]. The analysis of modulational instability associated with the propagation of intense laser pulses in a partially stripped, preformed plasma channel is undertaken which results the interplay between (anomalous) group velocity dispersion and self-phase modulation [14–16]. Very recently, Modulation instabilities (MI) in a one-dimensional chain configuration of a flexible mechanical metamaterial (flexMM) is reported, here flexMMs can be modeled by a coupled system of discrete equations for the longitudinal displacements and rotations of the rigid mass units [17]. In the variety of works, many authors discussed the modulation instability analysis for metamaterials with different linear and nonlinear effects [18–27]. In these works, some novel results are attained to understand the modulation instability in metamaterials. In addition, study on soliton dynamics in negative index optically metamaterials have been addressed by numerous researchers through pioneering works [26–38]. Particularly, with the presence of pseudo-quintic nonlinearity, self-steepening effect and delayed Raman response, controllable MI process have been addressed [39]. Moreover, as another application of optical metamaterials, ternary logic operation is achieved through employing mechanical metamaterials [40]. With the consideration of birefringent effects in metamaterial, cross phase modulation induced modulational instability analysis have been reported [41]. In [26], modulation instability influenced by nonlinear dispersion term in an optical metamaterial with the accompanying of coherent and partially coherent ultrashort pulses discussed in detail. In the realm of scientific exploration, where curiosity and innovation intertwine, a multitude of pioneering works have emerged, each unveiling the tantalizing novelty of optical soliton solutions across an array of mathematical models [27, 42–59]. Several studies contribute significantly to the field of nonlinear wave equations and soliton theory, but they also reveal specific gaps in understanding soliton dynamics, stability under varying conditions, and the influence of higher-order nonlinearities and dispersions. I aim to address these gaps by providing comprehensive solutions and stability analyses for a broader class of nonlinear equations, incorporating modern computational techniques, and exploring new physical phenomena [60–68]. Lump solutions with higher-order rational dispersion relations were provided [69]. While the focus was on lump solutions, the interplay between lump and soliton solutions under various conditions will be explored and breathers, rogue waves, and lump interactions for the Schrödinger–Hirota equation were discussed [70]. This research will be complemented by a detailed study of soliton interactions and stability in similar and extended models. These mesmerizing behaviour soliton pulse, seemingly defying the laws of dispersion and decay, have sparked a revolution in the world of fiber optic communication systems. Yet, amidst the brilliance of these discoveries, an equally crucial endeavor unfolds–a relentless quest to decipher the intricate tapestry of soliton stability [71–73]. In this paper, our research delves into the unique manifestations of modulation instability (MI) within negative index metamaterials, with a specific focus on the critical roles played by both intermodal and higher-order dispersions as described by the Fokas equation. These dispersions significantly influence the propagation and stability of waves in these materials, but this particular combination of factors is seldom explored in current scientific literature. By employing modulation instability as an analytical tool, we are able to gain deeper insights into the complex dynamics and stability characteristics of negative index metamaterials. Understanding these dynamics is crucial, as it sheds light on the behavior of these materials under various conditions, which in turn is vital for their potential applications in photonic and optical systems. Our approach not only advances theoretical understanding but also aids in the practical design and optimization of negative index metamaterials. The paper is organized as follows: section 1 discusses a generalized NLS model for the propagation of a few-cycle pulse in nonlinear metamaterials. Section 3 presents the modulation instability analysis. Sections 4 and 5 analyze the influence of intermodal dispersion and the interplay between amplitude and self-steepening parameters on MI in negative metamaterials. Section 6 explores the interplay between self-steepening parameters and nonlinear dispersion coefficients. Section 7 examines the interplay between intermodal dispersion and higher-order dispersion through numerical analysis, and the results are concluded in section 8.

2. The model

The generalized NLSE describingg the propagation of a few cycle pulse in nonlinear meta materials has the following form for the optical pulse envelope [74]

$$iE_z + i\alpha_1 E_t - \alpha_2 E_{tt} + \gamma|E|^2 E = i\lambda(|E|^2 E)_t + i\epsilon(|E|^2)_t E + \sigma_1(|E|^2 E)_{tt} + \sigma_2|E|^2 E_{tt} + \sigma_3|E|^2 E^*_{tt}, \tag{1}$$

where $E(z, t)$ represents the complex envelope of the electrical field, z and t are the propagation distance and time, respectively, while the parameters $\alpha_2, \gamma, \alpha_1, \lambda$ and ϵ represent the group velocity dispersion, cubic nonlinearity, intermodal dispersion, self-steepening, and nonlinear dispersion coefficients, respectively. Also, σ_i (with $i = 1, 2, 3$) are higher-order terms that appear in the context of metamaterials. The model (1) has garnered considerable attention for its pivotal role across diverse physical perspectives, particularly in the numerical exploration of soliton dynamics. It has illuminated crucial insights into the behavior of soliton parameters, encompassing a spectrum from numerically dark solitons to singular solitons, and from bright solitons to the intriguing W-shaped solitons. These findings underscore its significance in advancing our understanding of nonlinear phenomena in physics.

3. Stability analysis

Modulation instability is indeed a fundamental process in nonlinear systems, particularly in the context of wave propagation in various mediums, such as optics and fluid dynamics. It can lead to the spontaneous growth of waves with different frequencies, which can have important implications for applications such as signal processing, wave mixing, and nonlinear optics. It refers to the phenomenon where small perturbations on a continuous wave (cw) or constant-amplitude background can grow exponentially, leading to the spontaneous formation of a periodic pattern or a train of pulses. In the context of optics, modulation instability occurs when a continuous wave of light propagates through a nonlinear medium, such as an optical fiber or a nonlinear crystal. The interaction between the intensity of the wave and the nonlinear properties of the medium can lead to the growth of small fluctuations in the intensity. These fluctuations eventually become significant enough to cause the wave to break up into a series of pulses, forming a periodic pulse train. The period of the resulting pulse train is often related to the dispersion properties of the medium. Dispersion refers to the dependence of a medium's refractive index on the frequency of light. In the presence of anomalous dispersion (where shorter wavelengths travel slower than longer wavelengths) or normal dispersion (where longer wavelengths travel slower than shorter wavelengths), modulation instability can occur, leading to the formation of periodic structures with characteristic lengthscales determined by the dispersion properties. Modulation instability has important implications in various fields, including fiber optics communications, laser physics, and nonlinear optics. It can be both a challenging issue to mitigate in certain applications and a useful phenomenon for generating frequency combs or for studying nonlinear dynamics in complex systems. It's worth noting that the exact details of modulation instability, its characteristics, and its outcomes can vary depending on the specific physical system, the type of modulation instability (transverse or temporal), and the parameters involved.

We consider plane wave solution to be of the form $E(z, t) = \sqrt{P}\exp(i\omega z)$ with P being constant amplitude and Also $\omega = \gamma P$ The stability analysis of the PW solution is examined by taking including a small amplitude perturbation such that

$$E(z, t) = (\sqrt{P} + a(z, t))\exp(i\omega z) \tag{2}$$

where $a(z, t)$ represents small perturbations.

On substituting equations (2) into the model (1), and linearizing around the unperturbed plane wave yields

$$\begin{aligned} &ia_z + i\alpha_1 a_t - \alpha_2 a_{tt} + \gamma P(a + a^*) = i\lambda(2Pa_t + Pa_t^*) \\ &+ i\epsilon P(a_t + a_t^*) + \sigma_1(Pa_{tt}^* + 2Pa_{tt}) + \sigma_2 Pa_{tt} + \sigma_3 Pa_{tt}^* \end{aligned} \tag{3}$$

We consider the perturbations in the form of plane waves, $a(z, t) = \xi\cos(Kz - \Omega t) + i\zeta\sin(Kz - \Omega t)$ with real wave number K and complex eigen frequency Ω . Substituting this in equations (3) results that are obtained by solving the following set of homogeneous equations:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} \xi \\ \zeta \end{pmatrix} = 0 \tag{4}$$

where,

$$M_{11} = -K + \alpha_1\Omega - 3\lambda P\Omega - 2\epsilon P\Omega, \tag{5}$$

$$M_{12} = ((\sigma_1 + \sigma_2 - \sigma_3)P + \alpha_2)\Omega^2, \tag{6}$$

$$M_{21} = ((3\sigma_1 + \sigma_2 + \sigma_3)P - \alpha_2)\Omega^2 + 2\gamma P, \tag{7}$$

$$M_{22} = -K + \alpha_1\Omega + \lambda P\Omega \tag{8}$$

System will admit nontrivial solutions if its determinant is null, i.e., $det(M) = 0$, which leads, after some straightforward calculations, to the nonlinear dispersion relation in the following dispersion relation for eigen frequency

$$K^2 + AK + B = 0, \tag{9}$$

with

$$A = 2P\epsilon\Omega + 2P\lambda\Omega - 2\Omega\alpha_1, \tag{10}$$

$$\begin{aligned} B = &-\Omega^2(2P(\epsilon + \lambda)\alpha_1 - \alpha_1^2 - \Omega^2\alpha_2^2 + 2P\alpha_2(\gamma + \Omega^2(\sigma_1 + \sigma_3)) \\ &+ P^2(2\epsilon\lambda + 3\lambda^2 + \Omega^2(3\sigma_1^2 + \sigma_2^2 - \sigma_3^2) + 2\gamma(\sigma_2 - \sigma_3) + 2\sigma_1(\gamma + \Omega^2(2\sigma_2 - \sigma_3)))) \end{aligned} \tag{11}$$

The above nonlinear dispersion relation can be solved exactly to obtain,

$$K = \frac{-A \pm \sqrt{A^2 - 4B}}{2}; \tag{12}$$

The system remains stable under modulation of the indicated MI growth rate value is negative and this because of the vanishing long term of the growth rate $G = \sqrt{-K^2}$ exponentially over time. On the other hand for positive gain values, there is possibility of instabilities. Under such a condition, we know that MI will take place only when $A < 0, B < 0$, and the instability is a purely growing mode.

Figure 1 elegantly captures the essence of modulation within the metamaterial, allowing you to appreciate the harmonious interplay between spatial frequency and amplitude. In this visual representation, figure 1 offers an insightful view into the dynamic behavior of a metamaterial's gain spectrum as it interacts with changes in spatial frequency and amplitude. The graph serves as a window into the metamaterial's response to varying conditions, revealing the intricate relationship between these parameters and the resulting gain.

4. Influence of intermodal dispersion of negative metamaterials on MI

In this section we have explored Modulation instability (MI) in the context of negative index metamaterials (NIMs) can be influenced by intermodal dispersion. Negative index metamaterials are engineered materials that exhibit a negative refractive index, meaning they can have unusual optical properties not found in naturally occurring materials. Here we analyze Intermodal dispersion, which arises when different modes of propagation (such as different spatial or temporal modes) have different phase velocities, can affect the behavior of MI in such materials. Intermodal dispersion can result in complex dispersion relation (9) for different modes of propagation in negative index metamaterials. The dispersion relation equation (9) dictate how different frequencies and wave vectors are related. In the presence of negative refractive index, the dispersion relation can exhibit unusual behavior, leading to anomalous dispersion for certain frequency ranges. Intermodal dispersion can lead to mode coupling, where energy exchanges between different modes occur due to their differing phase

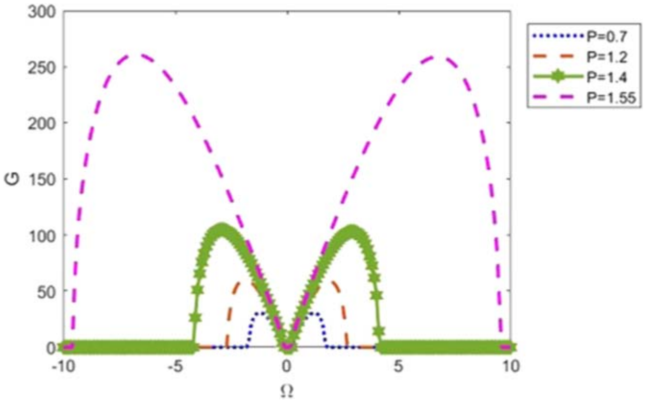
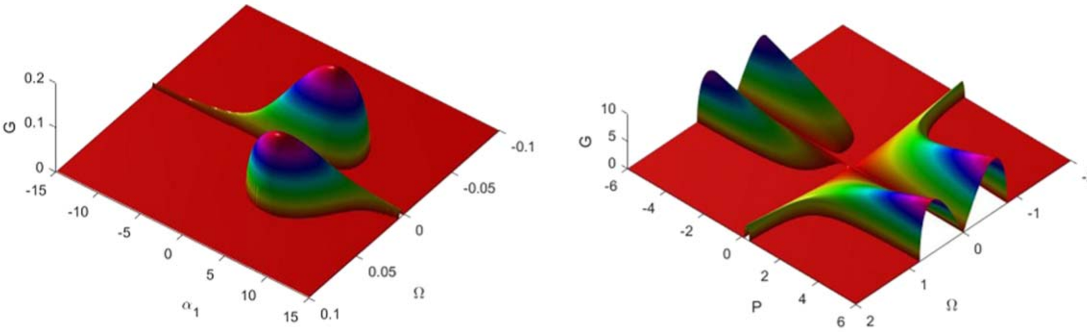


Figure 1. The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency with the parameter $\alpha_1 = 5, \alpha_2 = 25, \lambda = 0.7, \sigma_1 = 1.3, \sigma_2 = 25, \sigma_3 = 0.09, \gamma = 19$ and $\epsilon = 5$.



(a) (b)

Figure 2. (a) The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency and the parameter α_1 for $\alpha_2 = 11, \lambda = 0.2, P = 12, \sigma_1 = 17, \sigma_2 = 6, \sigma_3 = 25, \gamma = 0.01$ and $\epsilon = 0.1$. (b) The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency and the parameter P for $\alpha_1 = 1, \alpha_2 = 1, \lambda = 2, \sigma_1 = 0.4, \sigma_2 = 1, \sigma_3 = 5, \gamma = 0.1$ and $\epsilon = 0.9$.

velocities. Mode coupling can influence the MI process by introducing additional nonlinear interactions between the modes. This coupling can alter the threshold and characteristics of modulation instability. The presence of intermodal dispersion can lead to complex pulse dynamics and interactions in negative index metamaterials. As modulation instability evolves, the interplay between different modes and their dispersion characteristics can give rise to intricate pulse shapes and behaviors that differ from those in conventional materials. Figure 2 results intermodal dispersion in negative index metamaterials can significantly influence the behavior of modulation instability. It can affect dispersion relations, growth rates, mode coupling, pulse dynamics, and the overall characteristics of MI. Understanding these effects is crucial for both harnessing the potential of modulation instability in novel applications and for controlling or minimizing its impact in practical scenarios involving negative index metamaterials. The contour plots of the gain spectrum on intermodal dispersion parameter α_1 and frequencies with the variation of nonlinear dispersion parameter depicted in figure 3. It also shows that the maximum MI gain decreases and the MI gain band narrows as the nonlinear dispersion parameter ϵ increasing. At lower ϵ values, the waves interact in a way that allows for a wide range of frequencies to resonate and amplify—the MI gain band stretches far and wide, enveloping a diverse array of notes in its embrace. The most remarkable transformation is in the maximum MI gain in the metamaterial, the interplay between modes, nonlinear dispersion and intermodal dispersion creates a mesmerizing performance.

5. Interplay between amplitude and self steepening parameter of negative metamaterials on MI

From figure 4, it is observed that the interplay between the amplitude and the self-steepening parameter in negative metamaterials can have significant effects on modulational instability (MI). In negative metamaterials,

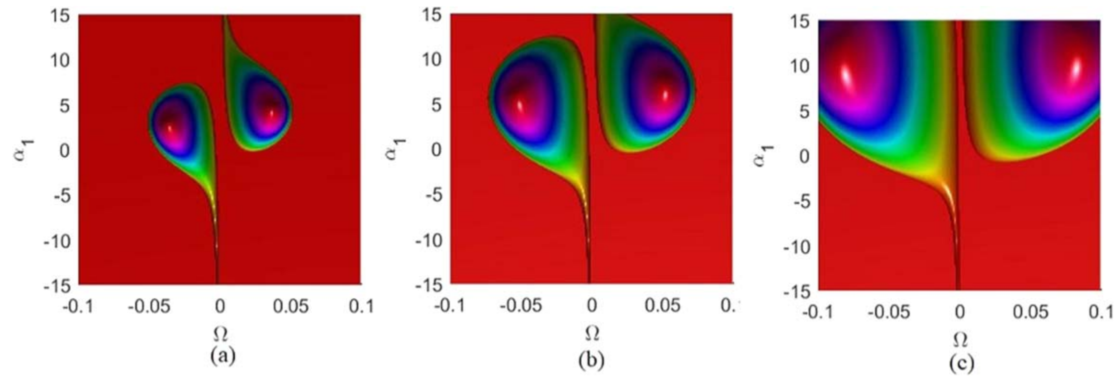


Figure 3. The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency and the parameter α_1 for $\alpha_2 = 11$, $\lambda = 0.2, P = 12, \sigma_1 = 17, \sigma_2 = 6, \sigma_3 = 25, \gamma = 0.01$, (*a*) $\epsilon = 0.1$, (*b*) $\epsilon = 0.3$ and (*c*) $\epsilon = 0.7$.

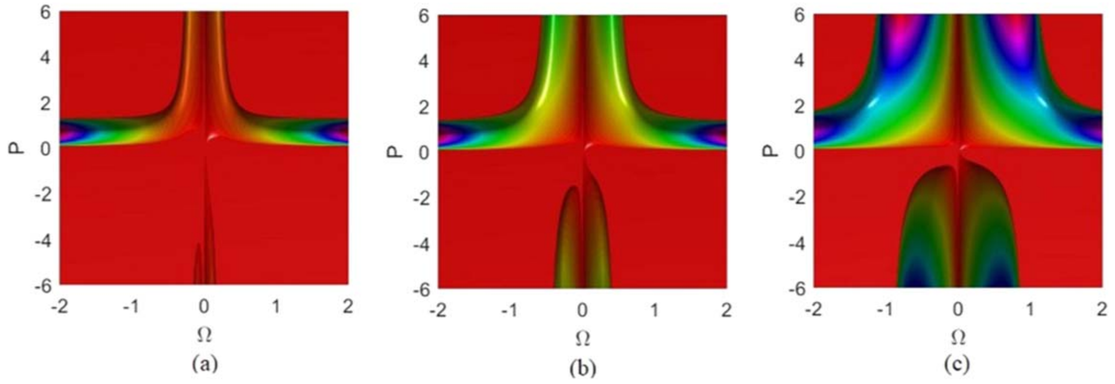


Figure 4. The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency and the parameter P for $\alpha_1 = 11$, $\alpha_2 = 1$ $P = 12, \sigma_1 = 17, \sigma_2 = 6, \sigma_3 = 25, \gamma = 0.01, \epsilon = 0.1$, (*a*) $\lambda = 0.5$, (*b*) $\lambda = 1$ and (*c*) $\lambda = 2$.

as the intensity of the wave increases, the nonlinear response becomes more pronounced. This can lead to stronger MI because higher-intensity waves are more susceptible to modulation. Larger amplitudes can result in more efficient energy transfer from the pump wave to sidebands during the MI process. The self-steepening parameter λ quantifies the degree to which the optical pulse steepens as it propagates through a medium. In negative metamaterials, λ can be tailored or enhanced due to their unique properties. A larger λ indicates stronger self-steepening effects, which can either enhance or suppress MI, depending on the specific conditions. If the negative metamaterial exhibits strong anomalous dispersion (where the group velocity is opposite to the phase velocity), higher amplitudes can lead to enhanced MI due to the interplay with self-steepening. As the pulse steepens, it can experience more significant spectral broadening, leading to the generation of new frequency components. In some cases, the self-steepening parameter may dominate, leading to pulse compression and suppression of MI, even at high amplitudes. This effect is more likely in situations where the negative metamaterial has weak or normal dispersion. Figure 4, showing the growth rate with respect to the amplitude, and the emergence of new modulation instability (MI) bands as the self-steepening parameter increases slowly, is consistent with the behavior observed in certain nonlinear optical systems, particularly in negative metamaterials with strong nonlinearity. This behavior can be understood in the context of the modulational instability phenomenon. In this case, Modulational instability occurs when small perturbations in the amplitude or phase of a continuous wave lead to the spontaneous generation of sidebands at different frequencies.

These sidebands can grow exponentially over time. The growth rate of these sidebands depends on various factors, including the amplitude of the wave and the dispersion properties of the medium. The self-steepening parameter also plays a crucial role. When the self-steepening parameter is small, its influence on the pulse dynamics is relatively weak. At low amplitudes, you may observe the standard MI behavior where a single MI band forms As the amplitude increases, the growth rate of this MI band may also increase, but it remains a single band. As you slowly increase the self-steepening parameter λ, which enhance the nonlinear effects in the

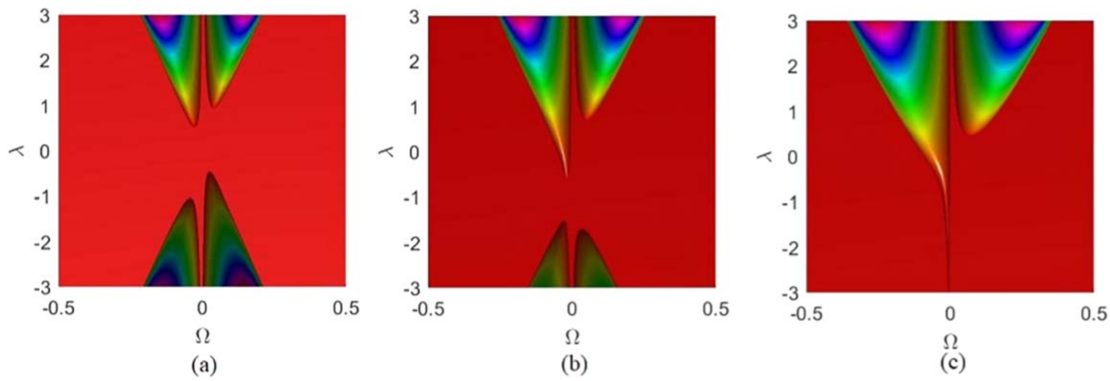


Figure 5. The instability growth rate $G = \sqrt{-K^2}$ plotted versus the perturbation wave frequency and the parameter λ for $\alpha_1 = 11$, $\alpha_2 = , \alpha_2 = 1, P = 12, \sigma_1 = 17, \sigma_2 = 6, \sigma_3 = 25, \gamma = 0.01$, (*a*) $\epsilon = 0.2$, (*b*) $\epsilon = 2$ and (*c*) $\epsilon = 8$.

medium. This means that as the pulse propagates, it becomes more susceptible to self-phase modulation (SPM) and self-steepening. SPM tends to broaden the spectrum of the pulse. When the self-steepening parameter becomes significant, it can lead to the formation of multiple MI bands. This occurs because the nonlinear effects are now strong enough to create multiple frequency components or sidebands as the pulse propagates through the medium. These new bands represent different frequency components resulting from the modulation of the original wave, and each band may have its own growth rate. The presence of multiple MI bands indicates a more complex MI spectrum. Depending on the system's parameters, these bands can be symmetric or asymmetric, and they may exhibit different growth rates. This complexity in the MI spectrum can have various practical implications, including the potential for the generation of new spectral components, the development of optical solitons, or the enhancement of nonlinear effects like supercontinuum generation.

6. Interplay between self steepening parameter and nonlinear dispersion coefficient of negative metamaterials on MI

Figure 5 gives the interplay between the self-steepening parameter λ and the nonlinear dispersion coefficient ϵ of negative metamaterials can significantly affect Modulational Instability (MI). Figure 5 predicts in negative metamaterials, which exhibit unique and controllable optical properties, this interplay can be particularly intriguing the self-steepening parameter and the nonlinear dispersion coefficient affect MI in negative metamaterials. Generally, the self-steepening parameter quantifies how the pulse steepens as it propagates through a nonlinear medium. It characterizes the strength of the nonlinear response. A larger λ indicates stronger self-steepening effects, leading to more pronounced nonlinear behavior. This can result in more efficient energy transfer from the pump wave to sidebands during MI. The nonlinear dispersion coefficient describes how the refractive index of a material changes with the intensity of light, indicating the strength of nonlinear dispersion. In negative metamaterials, ϵ can be engineered to have unique values, including negative values. Small values of nonlinear dispersion can lead to novel optical effects and can affect MI by introducing additional nonlinear contributions to the system. If both λ and ϵ have significant negative values, they can cooperate to enhance MI. Small values of ϵ can lead to the self-focusing of light, while λ amplifies the self-steepening effect. This combination can lead to the efficient generation of new spectral components during MI, resulting in the formation of multiple sidebands. In some cases, the self-steepening parameter and the nonlinear dispersion coefficient can have opposing effects. For instance, small values of ϵ might lead to self-defocusing, counteracting the self-focusing effect induced by λ. This can result in a more complex MI behavior where the net effect depends on the specific values of λ and ϵ. In the presence of strong self-steepening and small value of ϵ, might observe the formation of optical solitons. Hence the interplay between λ and ϵ can create conditions favorable for soliton formation. From figure 5 it is observed that increasing the nonlinear dispersion coefficient diminishes the new MI bands. This implies that as increase the nonlinear dispersion coefficient, the propensity for modulational instability to occur and create new spectral bands is reduced. In other words, a stronger nonlinear dispersion coefficient suppresses or hinders the formation of MI bands. When $\epsilon = 0.2$, we have four bands. At a relatively low nonlinear dispersion coefficient of 0.2, four MI bands are observed. This suggests that under these conditions, the medium is more conducive to modulational instability, resulting in the formation of these four bands. As the nonlinear dispersion coefficient is increased from 2 to 8, the lower-frequency MI bands (often referred to as 'bottom bands') gradually become weaker or disappear. This means that the increased nonlinear dispersion has a damping effect on these specific spectral components. The strength and number of

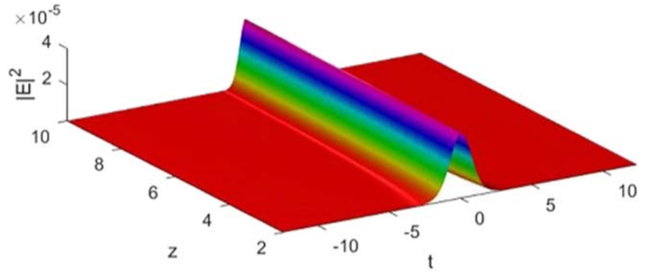


Figure 6. Spatiotemporal evolution of the densities as a consequence of development of MI for $\alpha_2 = 1, \lambda = 2, \sigma_1 = 0.01, \sigma_2 = 10,$ $\sigma_3 = 25, \gamma = 10, \lambda = 1$ and $\epsilon = 10$.

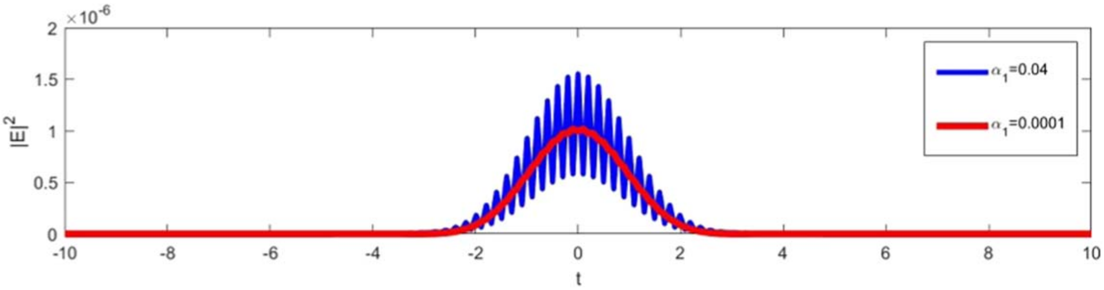


Figure 7. Cross sections of the density profiles for different values of intermodal dispersion when $\alpha_2 = 1$ $\lambda = 2, \sigma_1 = 0.01, \sigma_2 = 10,$ $\sigma_3 = 25, \gamma = 10, \lambda = 1$ and $\epsilon = 10$.

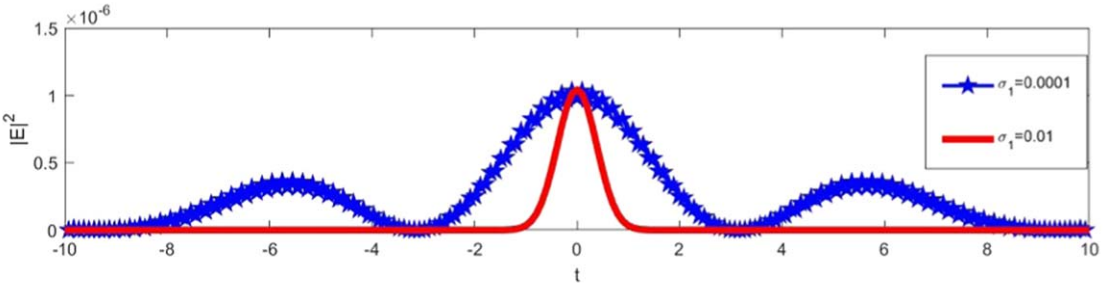


Figure 8. Cross sections of the density profiles for different values of higher order dispersion when $\alpha_1 = 0.0004, \alpha_2 = 1, \lambda = 2,$ $\sigma_2 = 10, \sigma_3 = 25, \gamma = 10, \lambda = 1$ and $\epsilon = 10$.

MI bands can be influenced by factors such as the nonlinear dispersion coefficient, the intensity of the incident light, and the properties of the nonlinear medium. Increasing the nonlinear dispersion coefficient may indeed reduce the occurrence or strength of MI bands, especially for the lower-frequency components.

7. Numerical analysis

In our quest for knowledge, we pit our analytical predictions against the unyielding challenge of direct numerical simulations on the generic equation (1). To achieve this objective, we employ the split-step Fourier method with specified initial conditions. To assess the precision of our digital experiment, we explore various combinations of spatial and temporal steps. Subsequently, we apply the split-step Fourier method while keeping t constant at 0 and z fixed, initializing the signal in the prescribed form

$$E(0,\, t) = n_0 + \zeta \cos(Kt), \tag{13}$$

where K is the perturbation wavenumber, initial density $n_0 = 1$ and $\zeta = 0.001$, so that $n_0 \ggg \zeta$. Under the above conditions, figure 6 shows the spatiotemporal evolution of the perturbed cw densities results solitons for the set of parameters $\alpha_2 = 1, \lambda = 2, \sigma_1 = 0.01, \sigma_2 = 10, \sigma_3 = 25, \gamma = 10, \lambda = 1$ and $\epsilon = 10$. Figures 7 and 8 present the results of a study that explores the effects of intermodal dispersion and second order nonlinear dispersion on the formation and properties of soliton solutions in a negative index metamaterial model. Figure 7 examines how intermodal dispersion affects the soliton solutions in which we can clearly observe intermodal dispersion typically arises in optical waveguides or fibers when multiple modes (different spatial patterns) of light propagate at different speeds. It can cause different components of a light pulse to spread out in time as they travel through the medium. When the intermodal dispersion coefficient is less than 0.0001, it suggests that the dispersion between different modes is relatively low. In such a case, the solitons are less affected by intermodal dispersion, and they can propagate smoothly with minimal distortion. This is desirable for maintaining the integrity of the soliton waveforms in optical fiber communication systems.

On the other hand, when the intermodal dispersion coefficient is greater than 0.0001, it implies that intermodal dispersion is relatively high. In this situation, the different modes of light travel at significantly different speeds, and this can lead to dispersion-induced spreading of the soliton pulses. The solitons may not be able to maintain their shape and amplitude as effectively, and dispersion in the solitary wave becomes more significant. This can limit the distance over which solitons can be used for data transmission.

Therefore, to achieve smooth propagation of solitons in optical fiber communication systems, it is essential to minimize intermodal dispersion, typically by carefully designing the optical fiber and the system parameters to ensure that the dispersion coefficient is kept low. Figure 8 appears to explore how second-order nolinear dispersion influences the soliton solutions. When the second order nonlinear dispersion coefficient is small (e.g., much less than 0.0001), it disrupts the balance required for soliton propagation. Strong second order nonlinear dispersion can lead to the breakup of solitons, introduce additional dispersion-induced effects, and make it challenging to maintain the integrity of soliton pulses. This can result in a less smooth propagation of solitons. When the second order nonlinear dispersion coefficient, becomes relatively large (e.g., greater than 0.01), it reduces the impact of second order nonlinear dispersion on soliton propagation. Solitons can propagate more smoothly, preserving their shape and amplitude over longer distances, provided that other parameters, including linear dispersion and nonlinearity, are well-balanced.

8. Conclusion

In summary, our study on the impact of intermodal dispersion and various parameters of negative metamaterials on modulation instability (MI) provides crucial insights for optical fiber communication systems. We have highlighted the significant roles of the interaction between amplitude and self-steepening parameters, as well as the interplay between self-steepening and nonlinear dispersion coefficients in affecting MI. Ensuring smooth soliton propagation necessitates minimizing intermodal dispersion, which can be achieved by carefully designing optical fibers and system parameters to maintain a low dispersion coefficient. Our research shows that second-order nonlinear dispersion is vital for soliton behavior. A small second-order nonlinear dispersion coefficient (e.g., much less than 0.0001) disrupts the balance needed for soliton propagation. In contrast, a strong second-order nonlinear dispersion can cause soliton breakup, introduce additional dispersion-induced effects, and hinder the maintenance of soliton pulse integrity. These findings emphasize the necessity of precise control over dispersion parameters to achieve reliable soliton propagation in optical fiber communication systems. We observe the specific behavior of MI in negative metamaterials depends on the material's dispersion properties, the wavelength of the incident light, and the initial pulse shape. Designing negative metamaterials with controlled dispersion and nonlinear properties allows to tailor the interplay between amplitude and self-steepening to achieve desired optical effects, such as the generation of new frequencies, pulse compression, or the formation of solitons. MI in negative metamaterials results on the engineering and control of both the self-steepening parameter and the nonlinear dispersion coefficient. Balancing various dispersion terms, nonlinearity, and other factors is crucial for optimizing soliton-based data transmission over optical fibers. By carefully designing and tailoring these parameters, can manipulate MI to achieve various optical effects, such as the generation of solitons, super-continuum generation, or spectral shaping, which have applications in optical communications, imaging, and signal processing.

Acknowledgments

S Saravana Veni acknowledges Amrita Vishwa Vidyapeetham, Coimbatore where this work was supported under Amrita Seed Grant (File Number: ASG2022141).

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

S Saravana Veni https://orcid.org/0000-0003-4359-3531
M S Mani Rajan https://orcid.org/0000-0003-0562-2469

References

[1] Agrawal G P 1987 *Phys. Rev. Lett.* **59** 880
[2] Agrawal G P, Baldeck P L and Alfano R R 1989 *Phys. Rev.* A **39** 3406
[3] Xu G, Chabchoub A, Pelinovsky D E and Kibler B 2020 *Phys. Rev. Research* **2** 033528
[4] Kraych A E, Agafontsev D, Randoux S and Suret P 2019 *Phys. Rev. Lett.* **123** 093902
[5] Tang D Y, Zhao L M, Wu X and Zhang H 2008 *Phys. Rev.* A **80** 023806
[6] Zakharov V E and Gelash A A 2013 *Phys. Rev. Lett.* **111** 054101
[7] Tamayama Y, Nakanishi T and Kitano M 2013 *Phys. Rev.* B **87** 195123
[8] Schiek R and Baronio F 2019 *Phys. Rev. Research* **1** 032036(R) R
[9] Klostermann H and Pierre Th 2000 *Phys. Rev.* E **61** 7034
[10] Kumar N, Pukhov A and Lotov K 2010 *Phys. Rev. Lett.* **104** 255003
[11] Nielsen A U, Xu Y, Todd C, Ferré M, Clerc M G, Coen S, Murdoch S G and Erkintalo M 2021 *Phys. Rev. Lett.* **127** 123901
[12] Erkintalo M, Hammani K, Kibler B, Finot C, Akhmediev N, Dudley J M and Genty G 2011 *Phys. Rev. Lett.* **107** 253901
[13] Drolet F and Viñals J 1997 *Phys. Rev.* E **56** 2649
[14] Sprangle P, Hafizi B and Peñano J R 2000 *Phys. Rev.* E **61** 4381
[15] Sun C, Waller L, Dylov D V and Fleischer J W 2012 *Phys. Rev. Lett.* **108** 263902
[16] Sprangle P, Krall J and Esarey E 1994 *Phys. Rev. Lett.* **73** 3544
[17] Demiquel A, Achilleos V, Theocharis G and Tournat V 2023 *Phys. Rev.* E **107** 054212
[18] Shackeerali M, Shafeeque Ali A K and Uthayakumar A 2022 *Optik* **256** 168660
[19] Mohanraj P, Kaliyamurthy a J, Rajesh Kumar a U and Sivakumar R 2023 *Optik* **277** 170699
[20] Lu X, Wu X, Xiang H, Shen J, Li Y, Li Y and Wang X 2022 *Int. J. Mech. Sci.* **221** 107166
[21] Mohanraj P, Sivakumar R and Vijayakumar M 2022 *Optik* **270** 169967
[22] Houwe A, Abbagari S, Saliou Y, Akinyemi L and Doka S Y 2023 *Wave Motion* **118** 103111
[23] Abbagari S, Houwe A, Akinyemi L, Saliou Y and Bouetou T B 2022 *Chaos Solitons Fractals* **160** 112255
[24] Fernandes M C, Mhatre S, Forte A E, Zhao B, Mesa O, Weaver J C, Bechthold M and Bertoldi K 2022 *Extreme Mechanics Letters* **51** 101549
[25] Megne L T, Tabi C B and Kofane T C 2020 *Phys. Rev.* E **102** 042207
[26] Akinyemi L, Houwe A, Abbagari S, Wazwaz A-M, Alshehri H M and Osman M S 2023 *Optik* **288** 171202
[27] Tariq K U, Wazwaz A-M and Javed R 2023 *Chaos, Solitons Fractals* **166** 112903
[28] Gao P, Zhang C, Ai J, Li G and Kang Y 2013 *Physica* A **392** 6506–11
[29] Xiang Y, Wen S, Dai X, Tang Z, Su W and Fan D 2007 *J. Opt. Soc. Am.* B **24** 12
[30] Pendry J B 2000 *Phys. Rev. Lett* **85** 2000
[31] Saha M and Sarma A K 2013 *Opt. Commun.* **291** 321–5
[32] Shalaev V M, Cai W, Chettiar U K, Yuan H-K, Sarychev A K, Drachev V P and Kildishev A V 2005 *Opt. Lett.* **30** 3356
[33] Silahli S Z, Walasik W and Litchinitser N M 2016 *J. Opt.* **18** 054010
[34] Sylvere A S, Justin M, David V, Joseph M and Betchewe G 2021 *Waves Random Complex Medium* **33** 2
[35] Wen S, Wang Y, Su W, Xiang Y, Fu X and Fan D 2006 *Phys. Rev.* E **73** 036617
[36] Xiang Y, Wen S, Dai X and Fan D 2010 *Phys. Rev.* E **82** 056605
[37] Xiang Y, Dai X, Wen S and Fan D 2011 *J. Opt. Soc. Am.* B **28** 908
[38] Yanga R, Min X, Tian J and Zhang W 2016 *Eur. Phys. J.* D **70** 39
[39] Zhang H, Wu J, Fang D and Zhang Y 2021 *Sci. Adv.* **7** 2021
[40] Zhou W, Su W, Cheng X, Xiang Y, Dai X and Wen S 2009 *Opt. Commun.* **282** 1440–7
[41] Zhuo H, Wen S, Dai X, Hu Y and Tang Z 2007 *Appl. Phys.* B **87** 635–41
[42] Kaur L and Wazwaz A-M 2018 *Optik* **174** 447–51
[43] Hasegawa A 2004 *J. Opt.* **33** 145–56
[44] Li Z and Zhu E 2023 *J. Opt.* **53** 1302–8
[45] Nandy S and Lakshminarayanan V 2015 *J. Opt.* **44** 397–404
[46] Aljahdaly N H, El-Tantawy S A, Wazwaz A-M and Ashi H A 2021 *Romanian Reports in Physics* **15** 971–83
[47] Thi T N and Van L C 2023 *J Opt* **52** 2296–305
[48] Mathanaranjan T, Mani Rajan M S, Veni S S, Yildirim Y *et al* 2024 *Ukrainian Journal of Physical Optics* **25** S1003–16
[49] Dowluru R K and Bhima P R 2011 *J. Opt.* **40** 132–42
[50] Wenjun L, Yujia Z, Wazwaz A M and Qin Z 2019 *Appl. Math. Comput.* **361** 325–31
[51] Kopçasiz B and Yaşar E 2023 *J. Opt.* **52** 1513–27
[52] Sun B and Wazwaz A-M 2018 *Commun. Nonlinear Sci. Numer. Simul.* **64** 1–13
[53] Wang S 2023 *J. Opt.* **52** 1602–7
[54] Seadawy A R 2014 *Computers and Mathematics with Applications* **67** 172–80
[55] Seadawy A R, Rizvi S T R, Ali I, Younis M, Ali K, Makhlouf M M and Althobaiti A 2021 **53** 172
[56] S Saravana Veni *et al* 2024 *Phys. Scr* **99** 085225
[57] Zhao Y H, Mathanaranjan T, Rezazadeh H, Akinyemi L and Inc M 2022 *Results in Physics* **43** 106083
[58] Mathanaranjan T *et al* 2022 *Opt Quant Electron* **54** 271
[59] Abdel-Gawad H I 2023 *Opt Quant Electron* **55** 298
[60] Inc M, Yusuf A, Aliyu A I and Hashemi M S 2018 *Eur. Phys. J. Plus* **133** 168
[61] Elbrolosy M E 2021 *Phys. Scr.* **96** 125275
[62] Hamali W, Zaagan A A and Zogan H 2024 *AIMS Mathematics* **9** 14913–31
[63] ur Rahman M, Sun M and Boulaaras S 2024 *Bound Value Probl.* **2024** 15

[64] Banikhalid M, Azmi A, Alquran M and Ali M 2024 *Nonlinear Engineering* **13** 20240005
[65] slam S M R 2024 *Sci. Rep.* **14** 11428
[66] Aslan E C and Mustafa Inc 2017 *Waves Random Complex Medium* **27** 594–601
[67] Poppenberg M, Schmitt K and Wang Z Q 2002 *Calc. Var.* **14** 329–44
[68] Manju K and Kumar M 2022 *Phys. Scr.* **97** 125204
[69] Ma W-X and Zhang L 2020 *Pramana-J. Phys.* **94** 43
[70] Ahmad A, Seadawy A R, Ahmed S and Rizvi S T R 2023 *Opt. Quantum Electron.* **55** 730
[71] Veni S S, Rajan M S M, Tabi C B and Kofané T C 2024 *Phys. Scr.* **99** 025202
[72] Veni S and Rajan M S M 2023 *Opt. Quantum Electron.* **55** 107
[73] Tabi C B, Veni S, Wamba E and Kofané T C 2023 *Phys. Lett.* A **485** 129087
[74] Triki H and Kruglov V I 2023 *Opt. Commun.* **546** 129826