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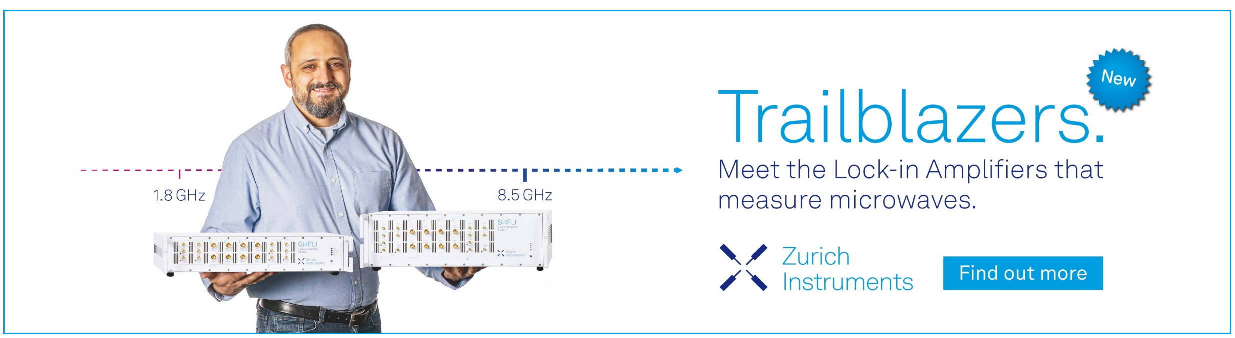

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Co-Even Domination Number of Complement of Some Graphs

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Abstract: The aim of this work is to determine the co-even domination number of complementary various graphs, as a ladder, lollipop, butterfly, jellyfish, helm, corona $\mathbf{C_n} \odot \overline{\mathbf{K_p}}$, fan, and double fan graph. Before that, the important properties of theco-even dominating set are mentioned from previous work.

Mathematical subject classification: 05C69

Keywords: Co-even domination number, complement co-even domination number.

INTRODUCTION

Domination is one of the most important concept in graph theory because it is an important way to solve many problems life. Recently, the concept of domination is interested in many researchers whether from researchers of mathematics, engineering, chemistry, or biology, and others. There are many kinds of domination are discussed [1-2], [5], and [14-15]. Also, it is introduced in many fields of graph theory as labeled graph [3-4] , topological graph [9-10], fuzzy graph [16-17] and the others. Where the researchers Omran and Shalaan [11] presented a new definition of domination which is called co-even domination and definition as follows, assume that G be a graph and D is a dominating set, the set D is called co-even dominating set if v has even neighbors for all $v \in V - D$**.** This set denoted by *CEDS*, also if it has no proper *CEDS*, then it is called minimal *CEDS* and denoted by *MCEDS*. Furthermore, if *MCEDS* has minimum cardinality, then it is called co-even domination number and denoted by $\gamma_{\text{coe}}(G)$. Again Omran and Shalaan in [12-13] determined this number on some graphs as a ladder, lollipop, butterfly, jellyfish, helm, corona, fan, and double fan graph and it is inverse. In this work, the complementary of the graphs mentioned above are been discussed. For more details about other concepts, the reader can see [6-8].

Proposition 1.1[11] *Let $G = (n, m)$ be a graph and D is a co-even dominating set, then*
1) *All vertices of odd or zero degrees belong to every co-even dominating set.*
2) $deg(v) \geq 2, for\ all\ v \in V - D.$
3) *If G is r- regular graph then* $\gamma_{coe}(G) = \begin{cases} n, & if\ r\ is\ odd \\ \gamma(G), & if\ r\ is\ even \end{cases}$
4) $\gamma\ (G)\ \leq \gamma_{coe}\ (G).$

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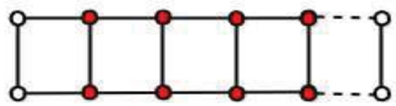
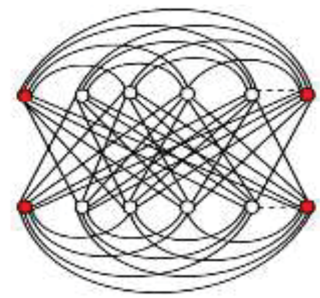
MAIN RESULTS

Proposition 2.1 *If* $G \equiv P_2 \times P_n$ *is ladder graph , then* $\gamma_{coe}(\bar{G}) = 4$.

Proof. Let G be a ladder graph of order $2n$, then all vertices of graph $\bar{G}$ have even degree except the four vertices which are the end vertices of the path P_n that have an odd degree (as an example, see Figure 2.1 (b)). Thus, according to proposition 1.1(1), these four vertices belong to every *CEDS* and they dominate all other vertices. Thus, $\gamma_{coe}(\bar{G}) = 4$.



(a) ladder graph L_n



(b) complement graph of (a)

FIGURE 1. Ladder graph with its complement

Proposition 2.2 *If* $G \equiv P_n + K_1$ *is a fan graph of order* $n+1$, *then*

$$\gamma_{coe}(\bar{G}) = \begin{cases} 3, & if\ n\ is\ odd; n > 3 \\ n+1, & if\ n \leq 3 \\ n-1, & if\ n\ is\ even; n \neq 2 \end{cases}$$

Proof. There are three cases are discussed as follows.

Case1. If n is odd and $n > 3$, then the degree of all vertices in $\bar{G}$ are even, except three vertices have odd degree one of them is the vertex represent K_1, since this vertex become the isolated in $\bar{G}$ and the other two vertices are the end vertices in the path P_n(as an example, see Figure 2.2(b)). Hence, according to proposition 1.1(1), $\gamma_{coe}(\bar{G}) = 3$.

Case2. if $n \leq 3$, then all vertices of $\bar{G}$ have an odd degree (as an example, see Figure 2.2(d)), so by proposition 1.1(1) $\gamma_{coe}(\bar{G}) = n+1$.

Case3. If n is even; $n \neq 2$, then the degree of all vertices of $\bar{G}$ are odd, except two vertices which are the end vertices in the path have even degree (as an example, see Figure 2.2(f)). Therefore, according to proposition 1.1(1), $\gamma_{coe}(\bar{G}) = n-1$.

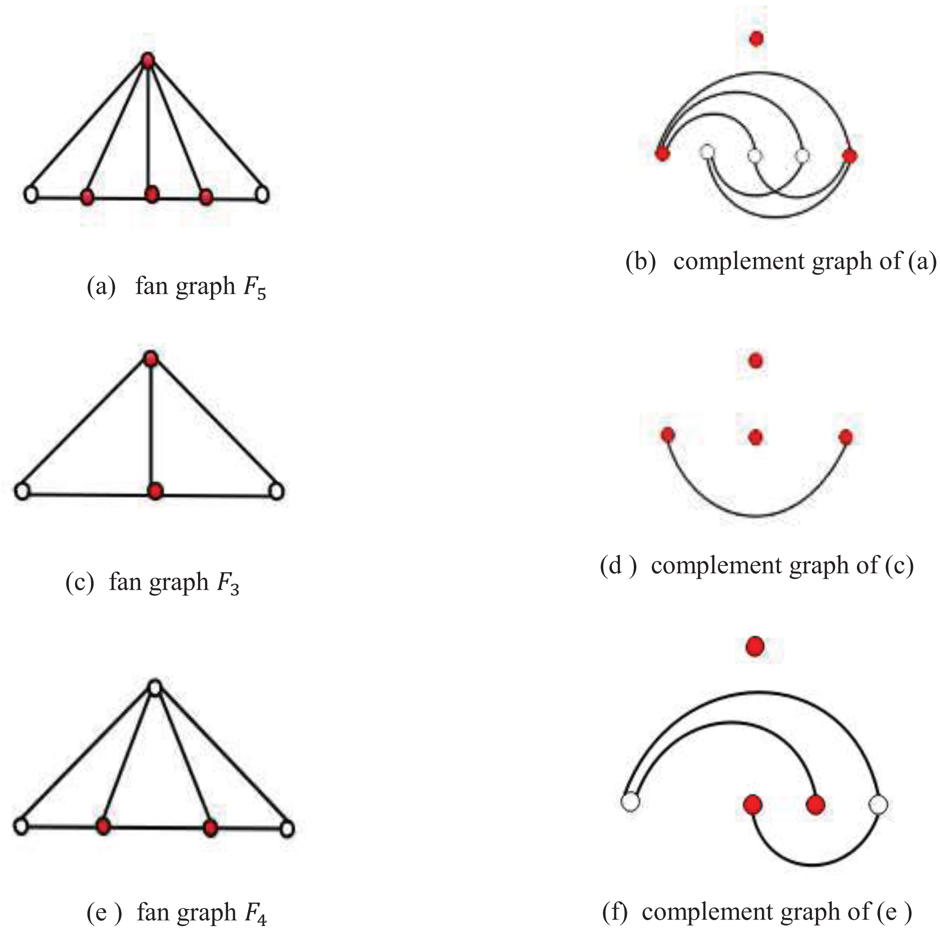


FIGURE 2. Fan graphs with its complement

Proposition 2.3 *If G is a double fan graph of order* $n+2$, *then*

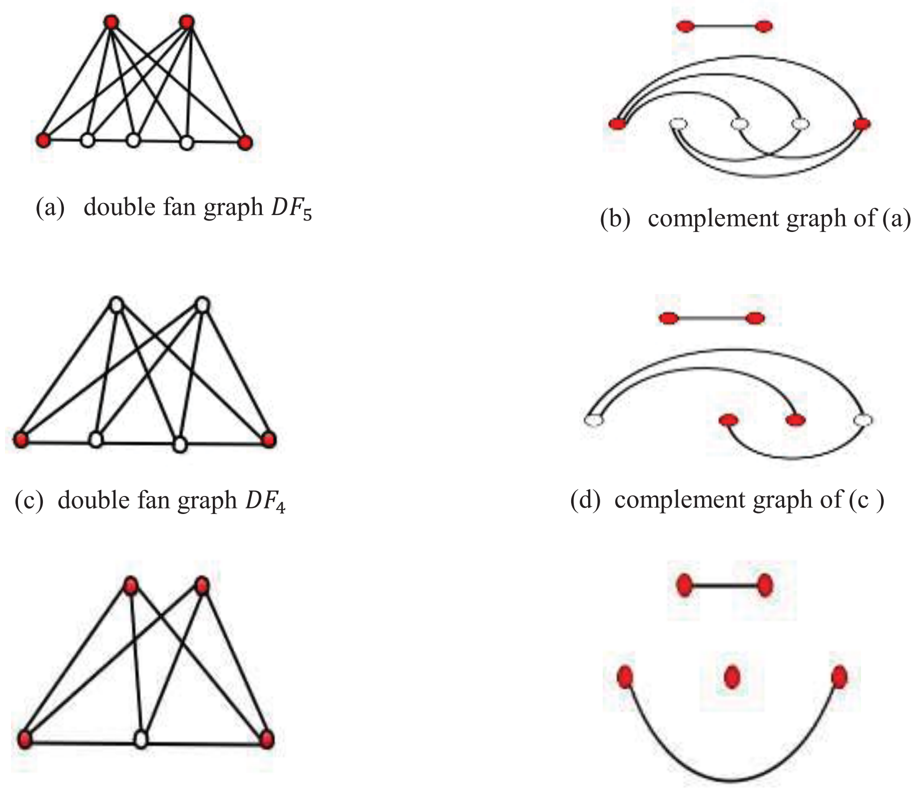
$$\gamma_{coe}(\bar{G}) = \begin{cases} 4, & if\ \ n\ is\ odd; n > 3 \\ n, & if\ n\ is\ even; n \neq 2 \\ n+2, & if\ n \leq 3 \end{cases}$$

Proof. There are three cases as follows.

Case1. If n is odd; $n > 3$, then all vertices of $\bar{G}$ have even degree, except two vertices which are end vertices of P_n and P_2 have an odd degree (as an example, see Figure 2.3(b)). Thus, according to proposition 1.1(1), $\gamma_{coe}(\bar{G}) = 4$.

Case2. If n is even; $n \neq 2$, then all vertices of $\bar{G}$ have odd degree, except two vertices which are end of P_n that have even degree (as an example, see Figure 2.3(d)). Hence, according to proposition 1.1(1) $\gamma_{coe}(\bar{G}) = n$.

Case3. If $n \leq 3$, then all vertices of $\bar{G}$ have an odd or zero degree, so $\gamma_{coe}(\bar{G}) = n + 2$. (as an example, see Figure 2.3(f)).



(a) double fan graph DF_5

(b) complement graph of (a)

(c) double fan graph DF_4

(d) complement graph of (c)

(e) double fan graph DF_3

(f) complement graph of (e)

FIGURE 3. Double fan graphs with its complement

Proposition 2.4 *If* $G \equiv C_n \odot \overline{K_p}$ *is corona graph of order* $n+p$*, then*

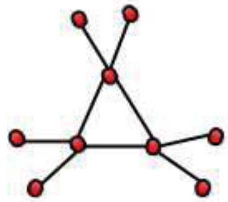
$$\gamma_{coe}(\bar{G}) = \begin{cases} np, & if\ n\ is\ odd\ and\ p\ is\ even \\ n, & if\ n\ and\ p\ are\ even \\ 2, & otherwise \end{cases}$$

Proof. There are three cases are discussed as follows.

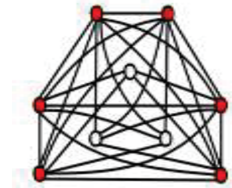
Case1. If n is odd and p is even, then the number of vertices in the graph $\bar{G}$ is odd. Thus, all pendants vertices of G become have odd degree in $\bar{G}$ and the vertices of the cycle in G become have even degree in $\bar{G}$. It is clear that the pendant vertices in G dominate all vertices in $\bar{G}$(as an example, see Figure 2.4(b)). Hence, $\gamma_{coe}(G) = np$.

Case2. If n and p are even, then the number of vertices in the graph $\bar{G}$ is even. Thus, the vertices of the cycle in G become have an odd degree in $\bar{G}$ and they dominate all other vertices which are even degree (as an example, see Figure 2.4(f)). Hence, $\gamma_{coe}(\bar{G}) = n$.

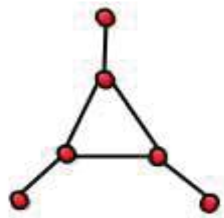
Case3. If n and p are odd or (n is even and p is odd), then the number of vertices in $\bar{G}$ is even, then all vertices in $\bar{G}$ have even degree (as an example, see Figure 2.4 (d)) and (h)). Therefore, $\gamma_{coe}(\bar{G}) = 2$.



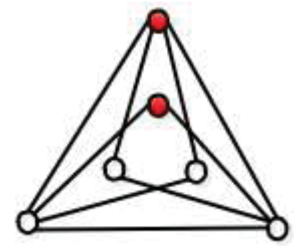
(a) corona graph $C_3 \odot \overline{K_2}$



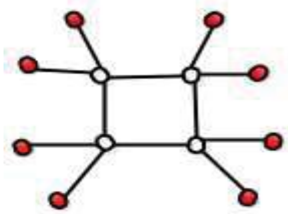
(b) complement graph of (a)



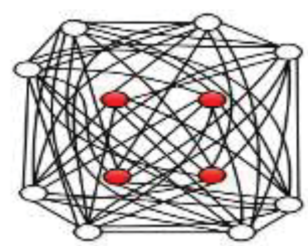
(c) corona graph $C_3 \odot K_1$



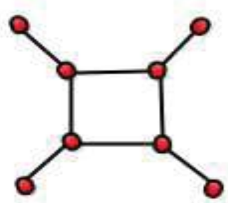
(d) complement graph of (c)



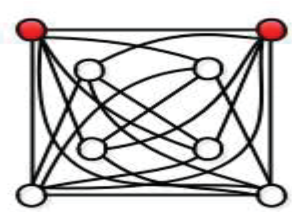
(e) corona graph $C_4 \odot \overline{K_2}$



(f) complement graph of (e)



(g) corona graph $C_4 \odot K_1$



(h) complement graph of (g)

FIGURE 4. Corona graphs with its complement

Proposition 2.5 if $G \equiv H_n$ *be helm graph of* $(2n-1)$ *vertices ,then*

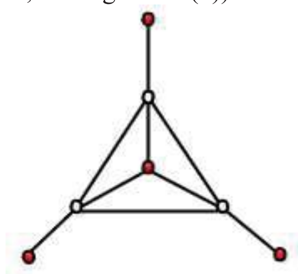
$$\gamma_{coe}(\bar{G}) = \begin{cases} n, & if\ n\ is\ even \\ n-1\ , & if\ n\ is\ odd \end{cases}$$

Proof.

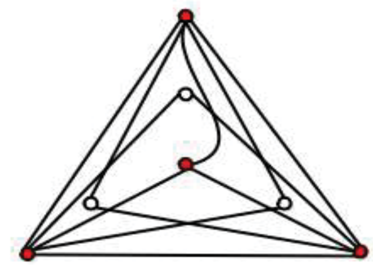
Let G be helm graph of order $(2n-1)$, Thus, all vertices in G become have an odd degree in the graph $\bar{G}$ except the center of the wheel which its degree depends on the order of the wheel W_n. Thus, all these vertices belong to every co-even dominating set according to proposition 1.1(1). The main vertex is the center of the wheel graph in G as mentioned above, so there are two cases as follows.

Case1. If n is even, then the degree of the center vertex of the wheel in G becomes has an odd degree in the graph $\bar{G}$ (as an example, see Figure 2.5(b)). Thus, it belongs to every co-even dominating set according to proposition 1.1(1). Hence, $\gamma_{coe}(\bar{G}) = n$.

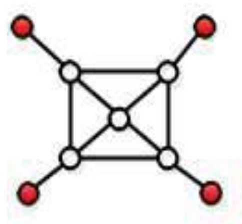
Case2. If n is odd, then the degree of the center vertex of the wheel in G becomes has even degree in the graph $\bar{G}$ (as an example, see Figure 2.5(d)). Hence , $\gamma_{coe}(\bar{G}) = n - 1$.



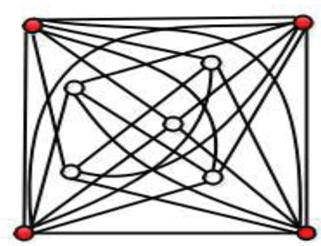
(a) helm graph H_4



(b) complement graph of (a)



(c) helm graph H_5



(d) complement graph of (c)

FIGURE 5. Helm graphs with its complement

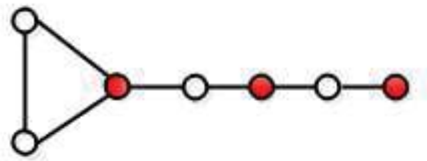
Proposition 2.6 *If G is a Lollipop graph* $L_{m,n}$ *of order* $m+n$ *, then*

$$\gamma_{coe}(\bar{G}) = \begin{cases} 2\,, & if\ m\ is\ odd\ and\ n\ is\ even \\ m\,, & if\ m\ is\ even\ and\ n\ is\ odd \\ n\,, & if\ m\,,n\ are\ even \\ m+n-2\,, & m,n\ are\ odd \end{cases} \quad where\ m \geq 3, n \geq 2$$

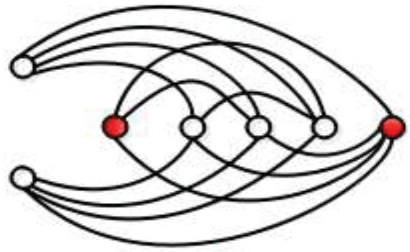
Proof. There are four different cases are obtained as follows.

Case1. If *m* is odd and *n* is even, then all vertices of $\bar{G}$ have an even degree except two vertices have odd degree one of them that represent the vertex which joins the complete graph with the edge that joined with path graph in G. The other vertex represent the pendant vertex in G. Thus, according to proposition 1.1(1) these two vertices belong to every *CEDS* (as an example, see Figure 2.6(b)). And they are dominate all other vertices. Thus, $\gamma_{coe}(\bar{G}) = 2$.

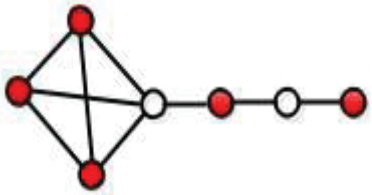
Case2. If *m* is even and *n* is odd, then every vertex of K_m in G become have an odd degree in $\bar{G}$ except one vertex which is mentioned in case1. Furthermore, all vertices of the path in G become have even degree in $\bar{G}$ except the pendant vertex in G has odd degree(as an example, see Figure 2.6(d)). Thus, according to proposition 1.1(1) and these vertices dominate all vertices in the graph $\bar{G}$. Thus, $\gamma_{coe}(\bar{G}) = m$.



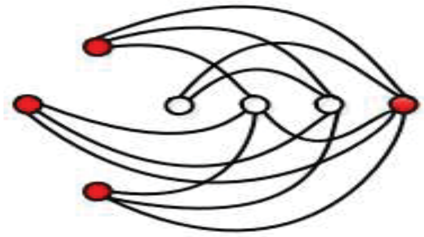
(a) lollipop graph $L_{3,4}$



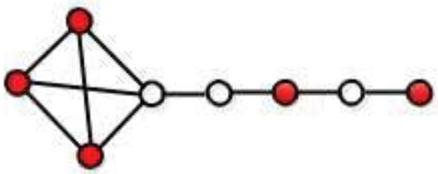
(b) complement graph of (a)



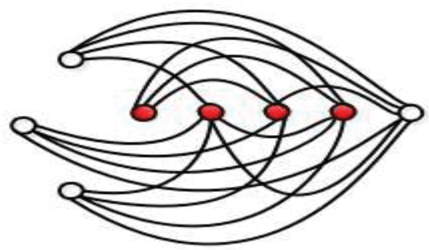
(c) lollipop graph $L_{4,3}$



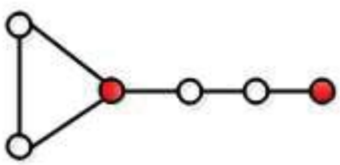
(d) complement graph of (c)



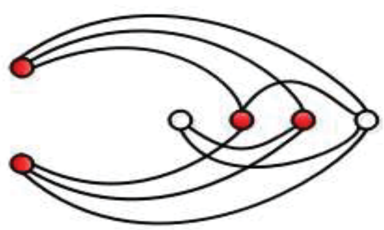
(e) lollipop graph $L_{4,4}$



(f) complement graph of (e)



(g) lollipop graph $L_{3,3}$



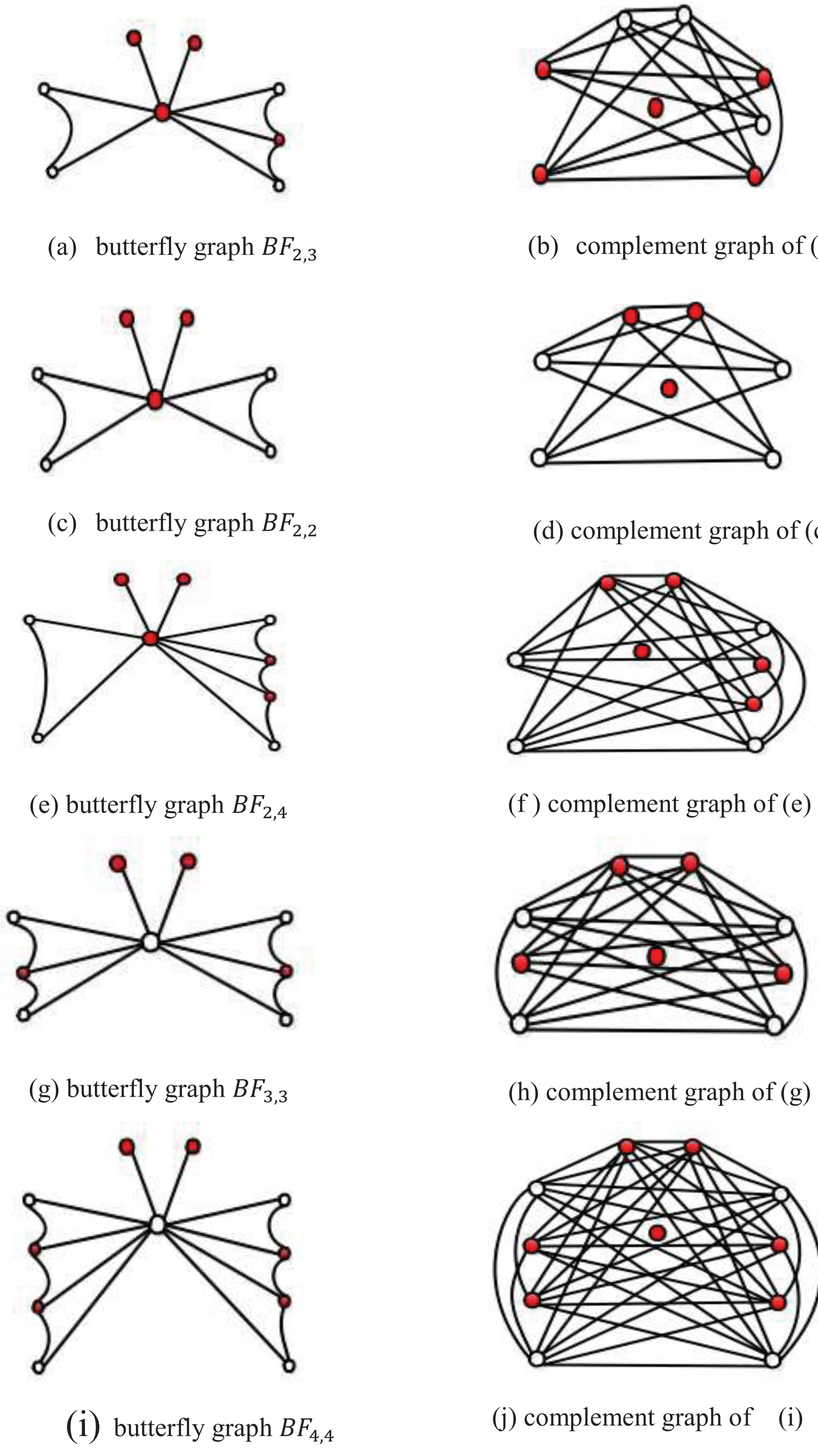
(h) complement graph of (g)

FIGURE 6. Lollipop graphs with its complement.

Case3. If m and n are even**,** then all vertices of K_m in G become have an even degree in $\bar{G}$ except one vertex which is mentioned in case1. Additionally, all vertices of P_n have an odd degree in G become have odd degree in $\bar{G}$ except the pendant vertex in G. It is clear that the vertices of odd degree belong to every co-even dominating set and they are dominating all vertices in the graph $\bar{G}$ (as an example, see Figure 2.6(f))Thus, $\gamma_{coe}(\bar{G}) = n$.
Case 4. If m and n are odd, then every vertices of $\bar{G}$ have odd degree except two vertices that mentioned in case1 (as an example, see Figure 2.6(h)). Thus, according to proposition 1.1(1), $\gamma_{coe}(\bar{G}) = m + n - 2$.

Proposition 2.7 *If G is a butterfly graph* $BF(m,n)$ $of\ order$ $(m + n + 3)$ *,then*

$$\gamma_{coe}(\bar{G}) = \begin{cases} 5\ , if\ m\ is\ even\ and\ n\ is\ odd\ or\ vice\ versa \\ n+1, if\ m, n\ are\ even; m=2\ and\ n \geq 2 \\ m+n-1, if\ m, n\ are\ even(odd); n, m \neq 2 \end{cases} \quad where\ m, n \geq 2.$$



(a) butterfly graph $BF_{2,3}$ (b) complement graph of (a)

(c) butterfly graph $BF_{2,2}$ (d) complement graph of (c)

(e) butterfly graph $BF_{2,4}$ (f) complement graph of (e)

(g) butterfly graph $BF_{3,3}$ (h) complement graph of (g)

(i) butterfly graph $BF_{4,4}$ (j) complement graph of (i)

FIGURE 7. Butterfly graphs with its complement.

Proof. There are three different cases are obtained as follows.

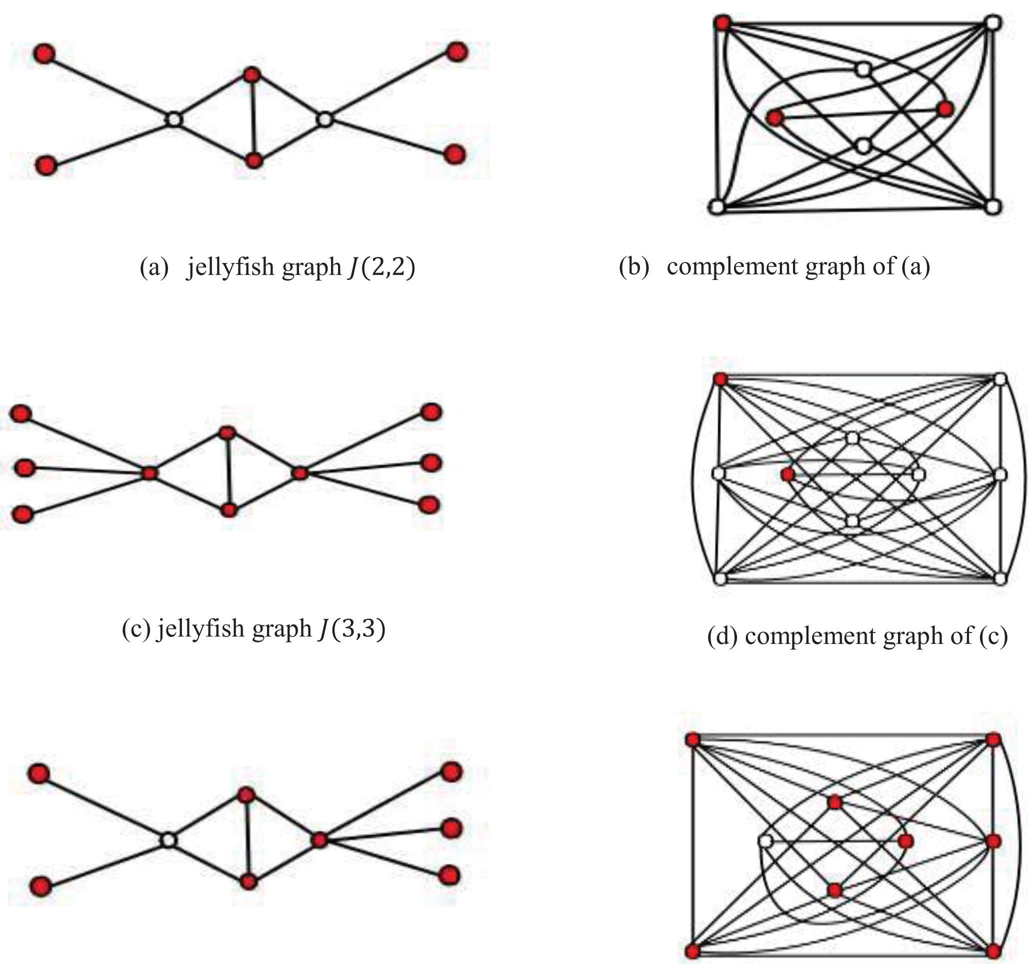
Case1. If either $m\ or\ n$ is even ,then all vertices of $\bar{G}$ have even degree except five vertices which are apex vertex and the end vertices of m and n in G become have odd degree in $\bar{G}$ Additionally, these five vertices dominate all other vertices of $\bar{G}$ (as an example, see Figure 2.7(b)). Thus, according to proposition 1.1(1) $\gamma_{coe}(\bar{G}) = 5$.

Case2. If m, n are even; $m = 2$,then every vertices of $\bar{G}$ have even degree except three vertices which are the pendent and apex of G have odd degree (as an example, see Figure 2.7(d) and (f)).So by proposition 1.1(1) these three vertices belongs to every co-even dominating set which are dominate all vertices, therefore $\gamma_{coe}(\bar{G}) = n + 1$.

Case3. If m, n are even(odd); $n, m \neq 2$,then all vertices have odd degree except the end vertices $of\ m$ and n have even degree which are four vertices (as an example, see Figure 2.7 (h) and (j)). so by proposition 1.1(1) every vertices of odd degree belong to every co-even dominating set. Therefore, $\gamma_{coe}(\bar{G}) = m + n - 1$.

Proposition 2.8 *If G is a jellyfish graph $J(m,n)$ of order $m + n + 4$,then*

$$\gamma_{coe}(\bar{G}) = \begin{cases} 3, & if\ m,n\ are\ even \\ 2, & if\ m,n\ are\ odd \\ m+n+3, & if\ either\ m\ or\ n\ is\ even \end{cases}$$



(a) jellyfish graph $J(2,2)$ (b) complement graph of (a)

(c) jellyfish graph $J(3,3)$ (d) complement graph of (c)

(e) jellyfish graph $J(2,3)$ (f) complement graph of (e)

FIGURE 8. Jellyfish graphs with its complement

Proof. Let G be jellyfish of order $(m + n + 4)$, then there exists three cases are discussed as follows.

Case1. If m and n are even, then all vertices of $\bar{G}$ have even degree except two vertices have odd degree which are even degree in G which are even degree in G. According to proposition 1.1(1) these two vertices not dominate all vertices in G. So, if add any vertex of m or n to the two vertices of odd degree, then the co-even dominating set is been obtained in $\bar{G}$ (as an example, see Figure 2.8(b)). Therefore, $\gamma_{coe}(\bar{G}) = 3$.

Case2. If m and n are odd, then all vertices of $\bar{G}$ have even degree. (as an example, see Figure 2.8(d)). Hence, $\gamma_{coe}(\bar{G}) = 2$.

Case3. If either m or n is even, then all vertices of $\bar{G}$ have odd degree except one vertex has even degree which is even degree in G (as an example, see Figure 2.8(f)). Hence, according to proposition 1.1(1), $\gamma_{coe}(\bar{G}) = m + n + 3$.

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